# INRIA Contribution
# 3rd AIAA Sonic-Boom workshop

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Inria informatics mathematics

# Scope of the presentation

**Near field cases:**

| | |
|---|---|
| bi-convex | provided-unstructured (not presented) |
| c608 | provided-unstructured and adaptive grids |

**Improvements from previous worskhops :**

$\Longrightarrow$ Only RANS computations

$\Longrightarrow$ Fully adaptive unstructured boundary layer

**General context**

- Tetrahedral meshes only
- Parallel computation 24 cores (48 threads), Xeon E7 , 128-512Gb
- Error estimate [Alauzet & Loseille, JCP 2008], [Loseille et al., JCP 2010]

  multi-scale (Lp) interpolation error Mach number

# INRIA Wolf Flow Solver

[Alauzet & Loseille, JCP 2012], [Menier et Al., AIAA 2014], [Frazza et al., AIAA 2018]

Compressible Turbulent Navier-Stokes flow solver:

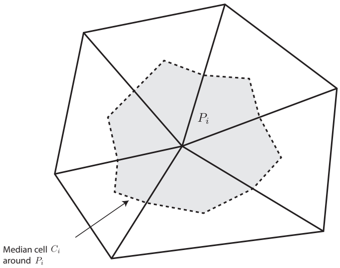
- Turbulence models: Spalart-Allmaras
- Full unstructured tetrahedra/hybrid meshes
- Mixed-Element-Volume Method:
  - Convective and source terms by Finite Volume method
  - Diffusive terms by Finite Elements method or Edge Based viscous formulation
- Vertex centered: low complexity
  - Median cells
  - Containment sphere cells
- Edge-based fluxes computation with a $2^{nd}$ order scheme using MUSCL reconstructions.
- Full differentiation with first order matrix.

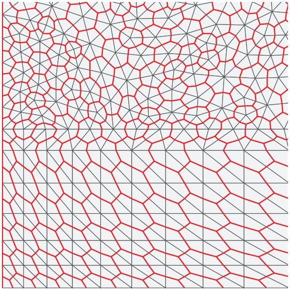
**Compressible Turbulent Navier-Stokes Equations:**

$$\frac{\partial W}{\partial t} + \nabla \cdot \mathcal{F}(W) = \mathcal{S}(W) + \mathcal{Q}(W)$$

Finite volume solution discretization: $W_i = \frac{1}{|C_i|} \int_{C_i} W \,\mathrm{d}\Omega$

$$|C_i| \frac{\partial W_i}{\partial t} = \int_{\partial C_i} \mathcal{F}(W)\,\mathrm{d}\Gamma + \int_{C_i} \mathcal{S}(W) + \mathcal{Q}(W)\,\mathrm{d}\Omega$$



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Mixed-Element-Volume Method (MEV):

- Convective terms by Finite Volume method: Upwinding
- Diffusive terms by Finite Element method: Elliptic

Vertex-centered: low complexity, edge structure, extrapolation

Solve on each FV cell: $|C_i| \frac{dW_i}{dt} + \mathbf{F}_i - \mathbf{S}_i - \mathbf{Q}_i = R_i$

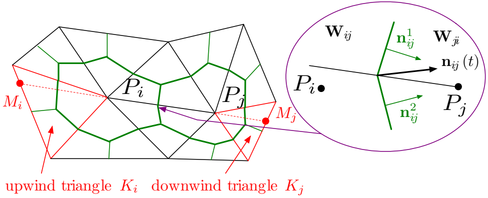
**Convective flux:**

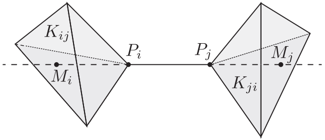
$$\mathbf{F}_i = \sum_{j \in \mathcal{V}(i)} F_{|\partial C_{ij}} \cdot \int_{\partial C_{ij}} \mathbf{n}_i \mathrm{d}\Gamma = \sum_{j \in \mathcal{V}(i)} \Phi_{ij}(W_i, W_j, \mathbf{n}_{ij})$$

- $\Phi_{ij}$ with Riemann solvers: mainly HLLC, other available Roe, Van Leer, ...

$2^{nd}$ order scheme using a MUSCL-like reconstruction at the cell interface:

- Low dissipation reconstruction: V3-scheme, V4-scheme, V6-scheme
  Combination of centered, upwind-downwind, nodal gradients
- Limiters: Dervieux/Piperno for low dissipation, all the usual for V3-scheme



Element-based classical FEM formulation for viscous terms

$$\int_{\partial C_i} \mathbf{S}(W_i)\cdot\mathbf{n}\,\mathrm{d}\Gamma \;=\; \sum_{P_j\in\mathcal{V}(P_i)}\int_{\partial C_{ij}} \mathbf{S}(W_i)\cdot\mathbf{n}\,\mathrm{d}\Gamma \;=\; -\sum_{K\ni P_i}\int_K \mathbf{S}(W_i)|_K\cdot\nabla\phi_i\,\mathrm{d}\mathbf{x}\,.$$

Turbulence model discretization:

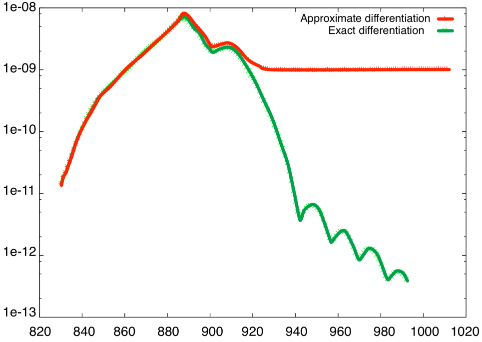
- Convective term: linear or non-linear convection
- Diffusive and dissipative terms: FEM formulation
- Production and destruction terms: FV source term formulation

**Accurate differentiation yields better results that approximate differentiation.**

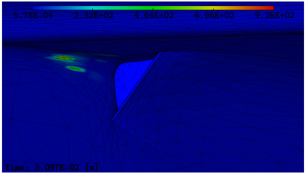
Convective fluxes $\frac{\partial \mathbf{F}_i^n}{\partial W_i}$ : Full first order differentiation (no MUSCL, no limiter). Differentiate HLLC solver !

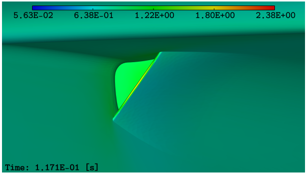
Viscous fluxes $\frac{\partial \mathbf{S}_i^n}{\partial W_i}$ : Direct finite elements differentiation. Do not use spectral radius approximations !

Turbulent source terms $\frac{\partial \mathbf{Q}_i^n}{\partial W_i}$ : Care for non-linearities.



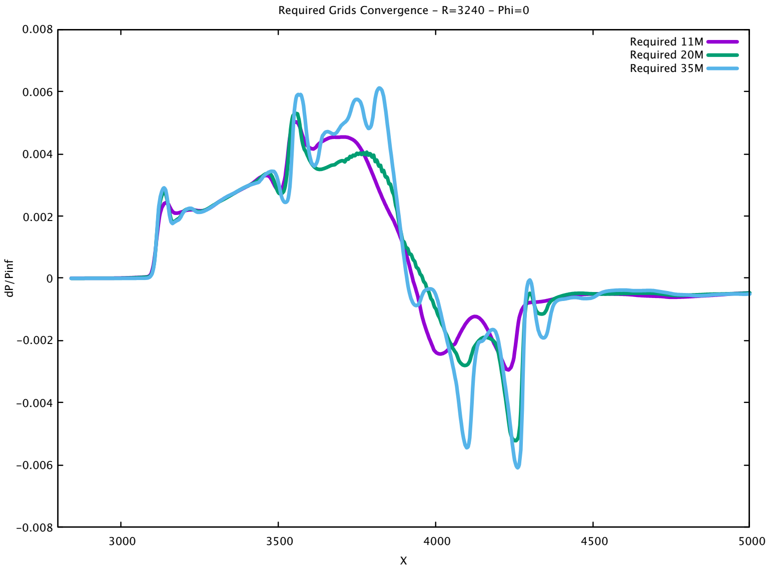
- All case are initialised with first order solution and then second order
- All computations are using 4-th order dissipation terms.
- Issues with the boundary condition (ECS)




- 4 orders of magnitude reduction in the residual
- Limited to 30 million nodes grids

# Required grids convergence at $\Phi = 0, 90, 180$



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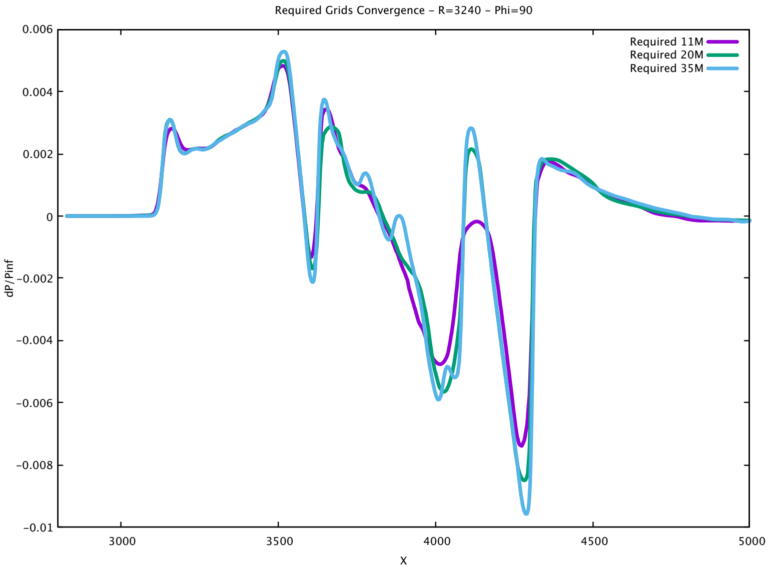

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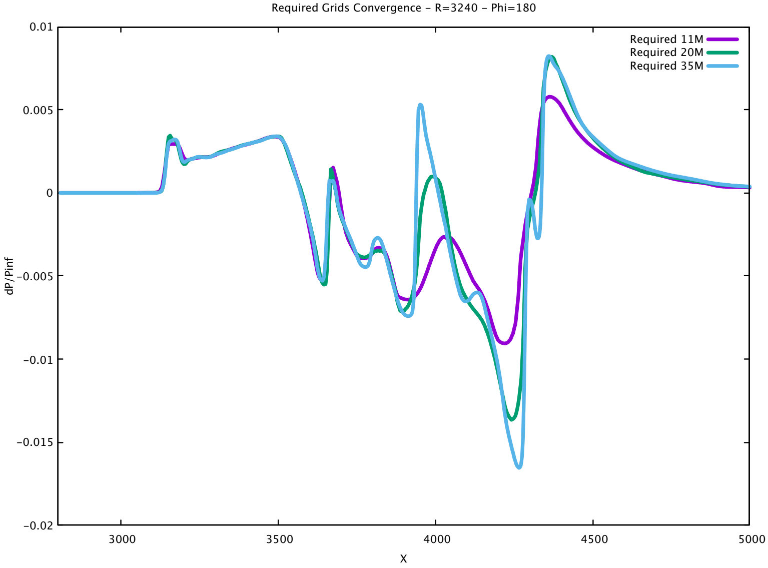


# Metric-based Anisotropic Mesh Adaptation

## Feature-based anisotropic mesh adaptation

Deriving the **best mesh** to compute the characteristics of a given solution **w**

[Tam et al., CMAME 2000], [Pain et al., CMAME 2001], [Picasso, SIAMJSC 2003], [Formaggia et al., ANM 2004], [Bottasso, IJNME 2004], [Li et al., CMAME 2005], [Frey and Alauzet, CMAME 2005], [Gruau and Coupez, CMAME 2005], [Huang, JCP 2005], [Compere et al., 2007], ...

## Multiscale anisotropic mesh adaptation

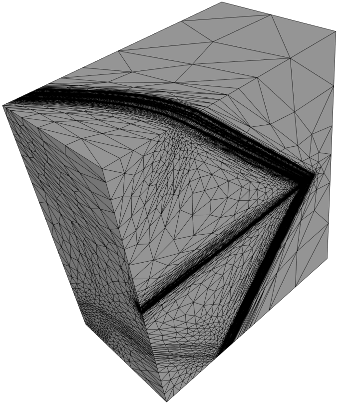
[Loseille et al., AIAA 2007], [Alauzet, IJNMF 2008]

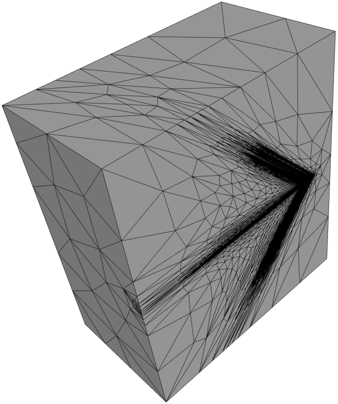
- Optimal control of the interpolation error in $L^p$ norm :

$$\|W - \Pi_h W\|_{L^p(\Omega_h)}$$

- Solve this problem in the continuous framework $\Longrightarrow \quad \mathcal{M}_{L^p}(H_W)$
- Highly anisotropic meshes
- Capture all scales of the flow
- Global $2^{nd}$ of mesh convergence for the mesh adaptation process
- Early capturing property: asymptotic convergence is reached faster

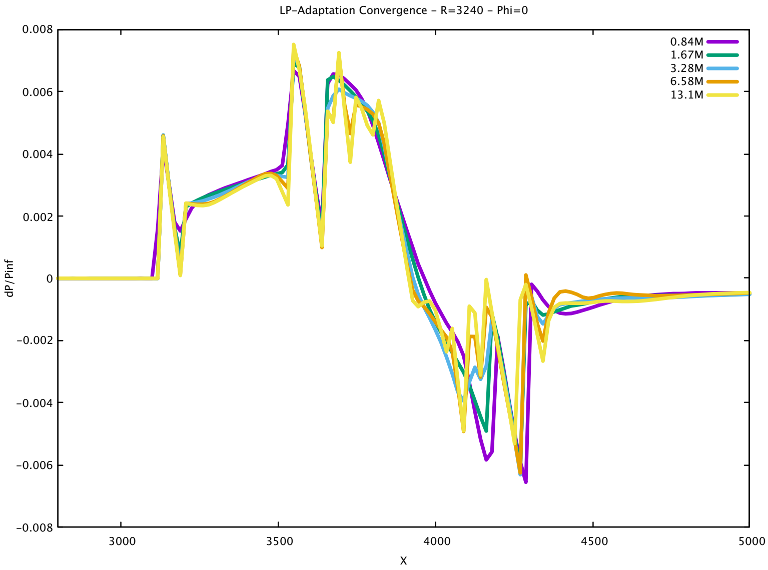
# Hessian correction





- Hessian anisotropic diffusion for $R > 4000$
- 5 adaptations are performed at fixed complexity from 200000 to 1280000

# L2-Adapted grids convergence at $\Phi = 0, 90, 180$



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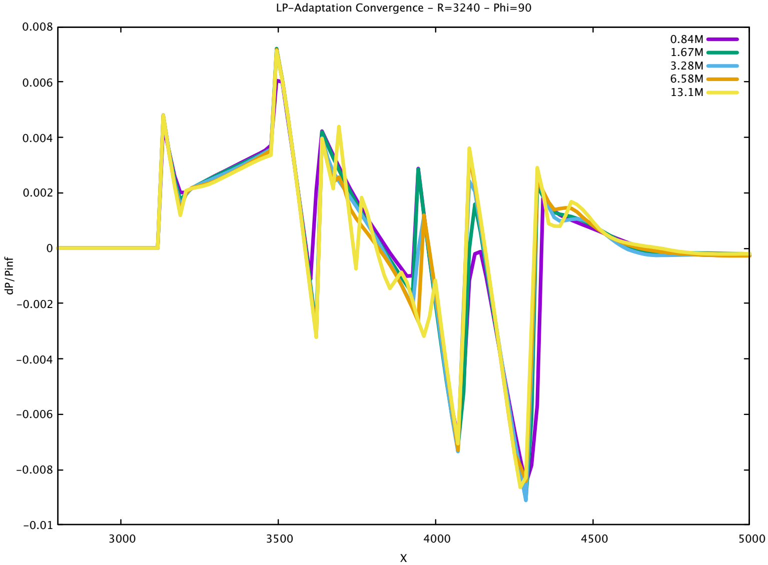

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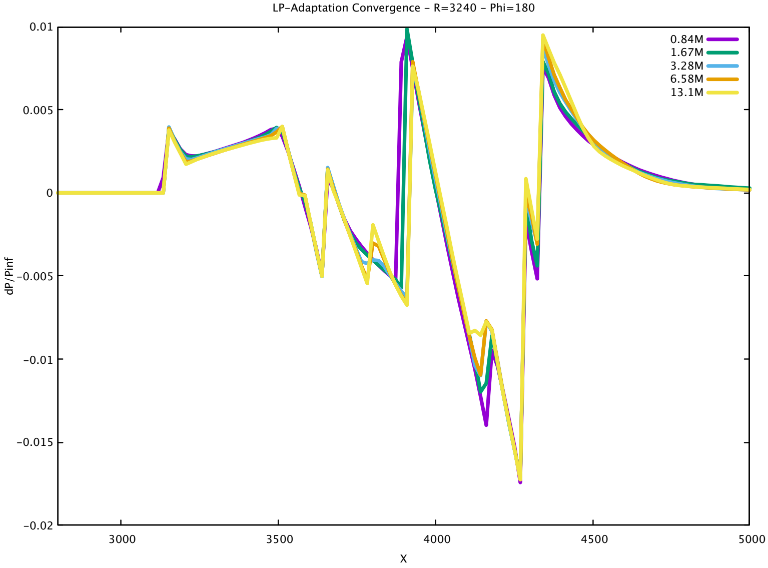
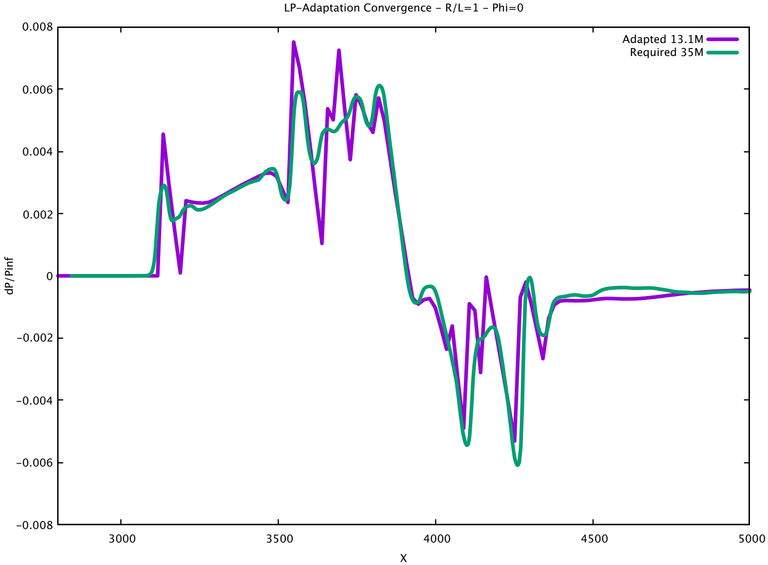
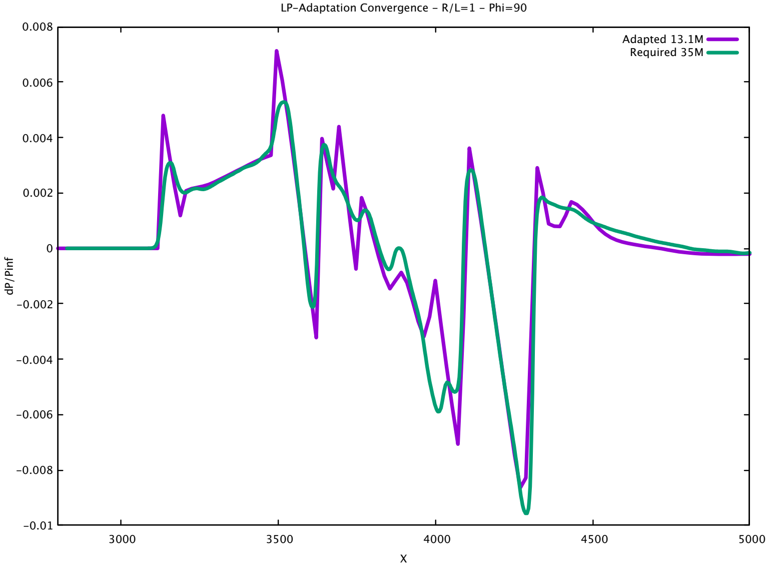


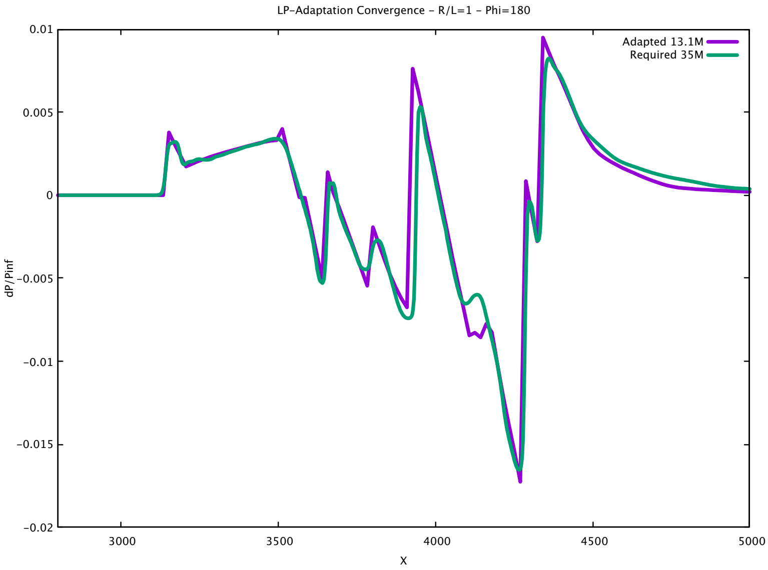


LP–Adaptation Convergence – R/L=1 – Phi=0
Adapted 13.1M
Required 35M
dP/Pinf
X
0.008
0.006
0.004
0.002
0
−0.002
−0.004
−0.006
−0.008
3000
3500
4000
4500
5000



LP-Adaptation Convergence – R/L=1 – Phi=90
Adapted 13.1M
Required 35M
dP/Pinf
X
0.008
0.006
0.004
0.002
0
−0.002
−0.004
−0.006
−0.008
−0.01
3000
3500
4000
4500
5000

# Comparison Adapted/Required at $\Phi = 0, 90, 180$



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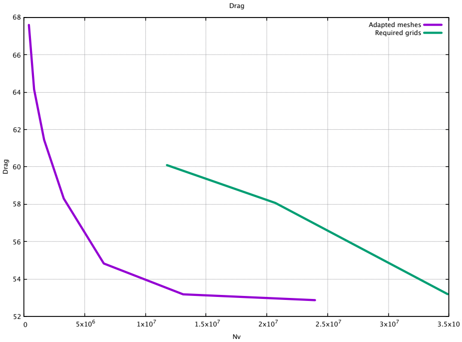
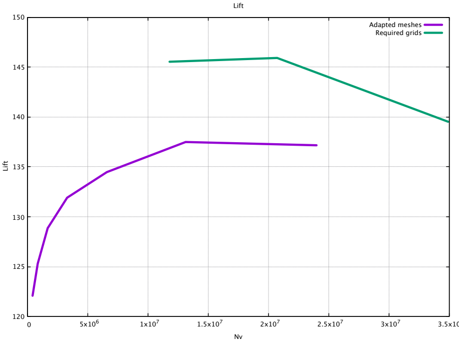
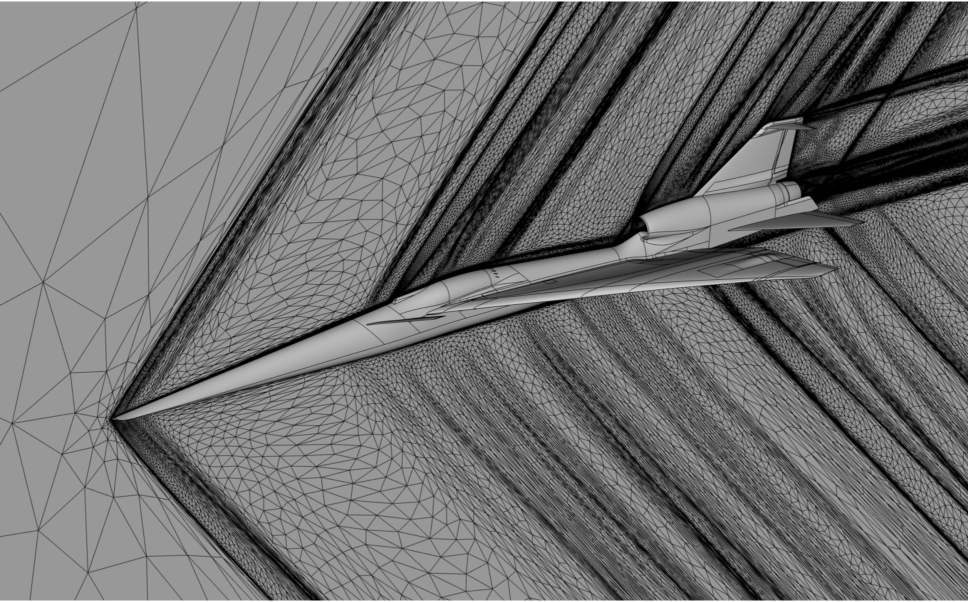


# Best effort adapted mesh : 11.3 M



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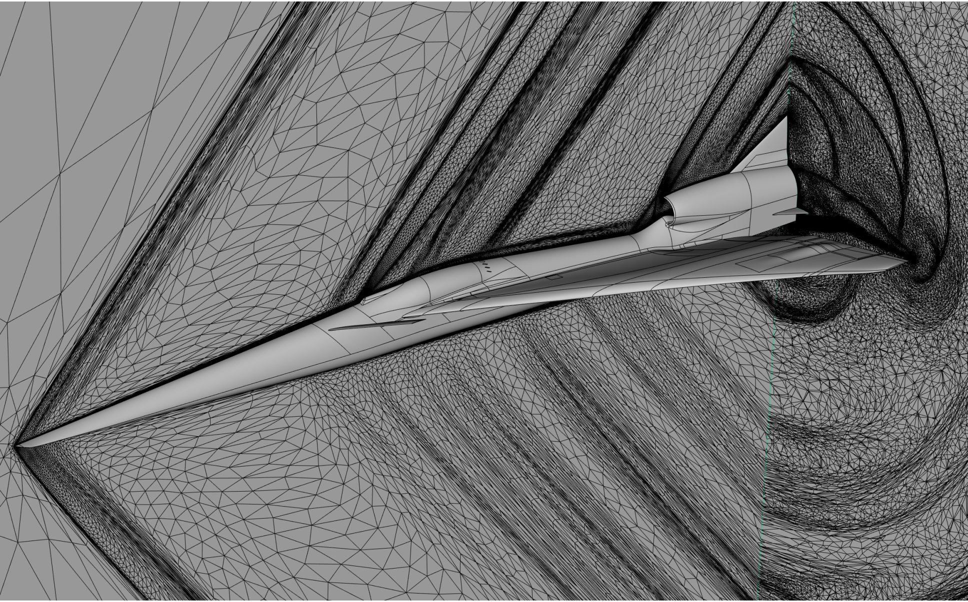

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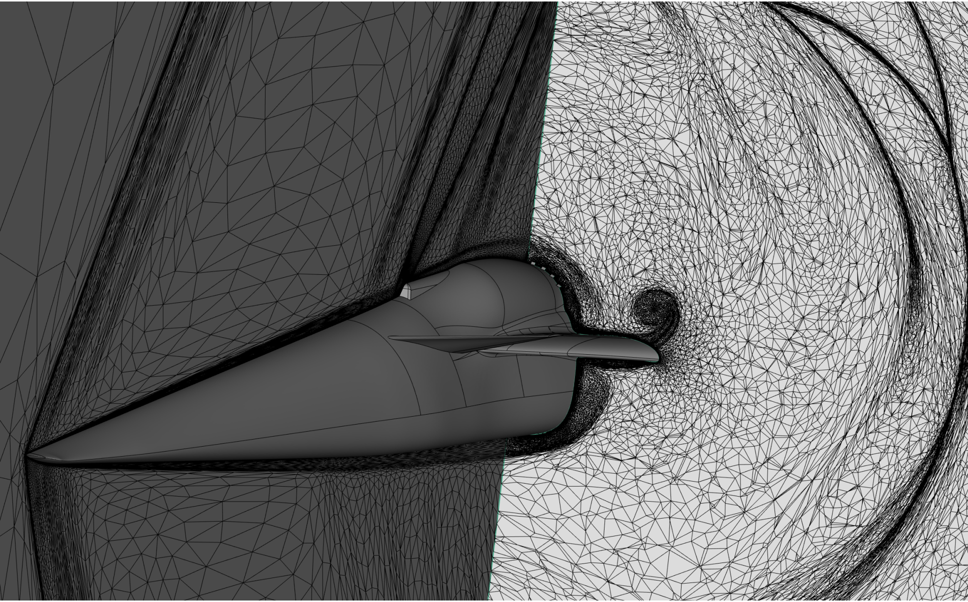

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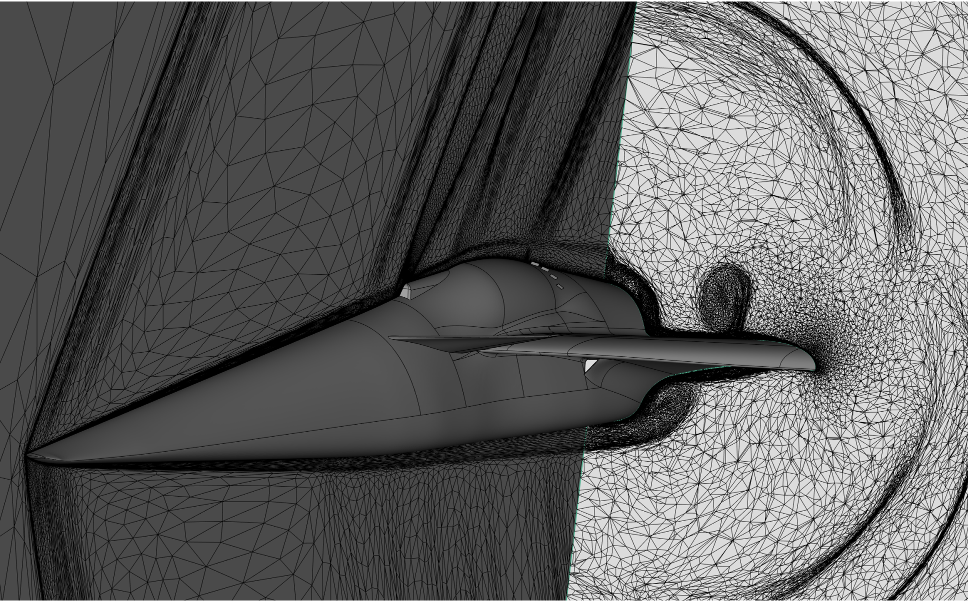

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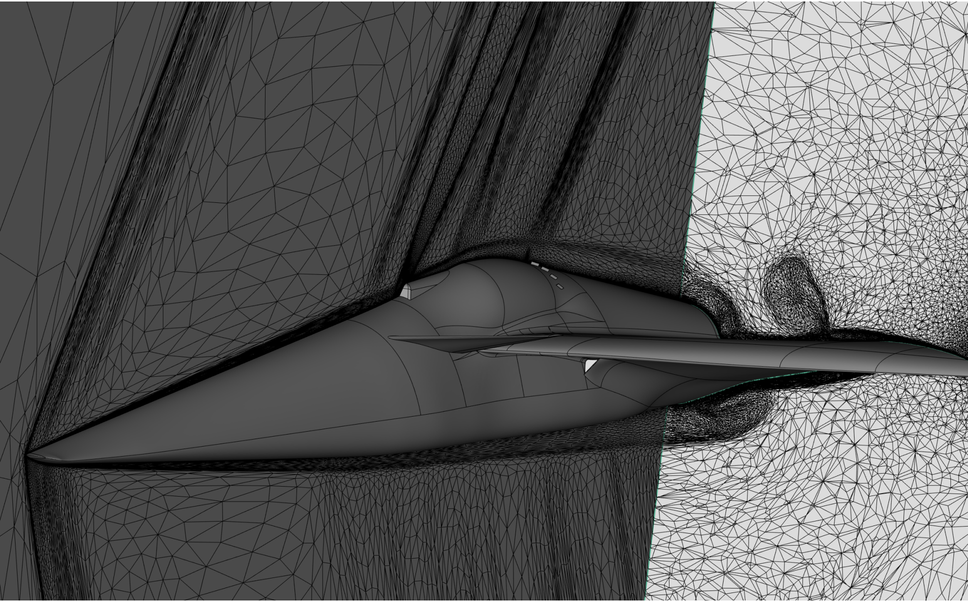

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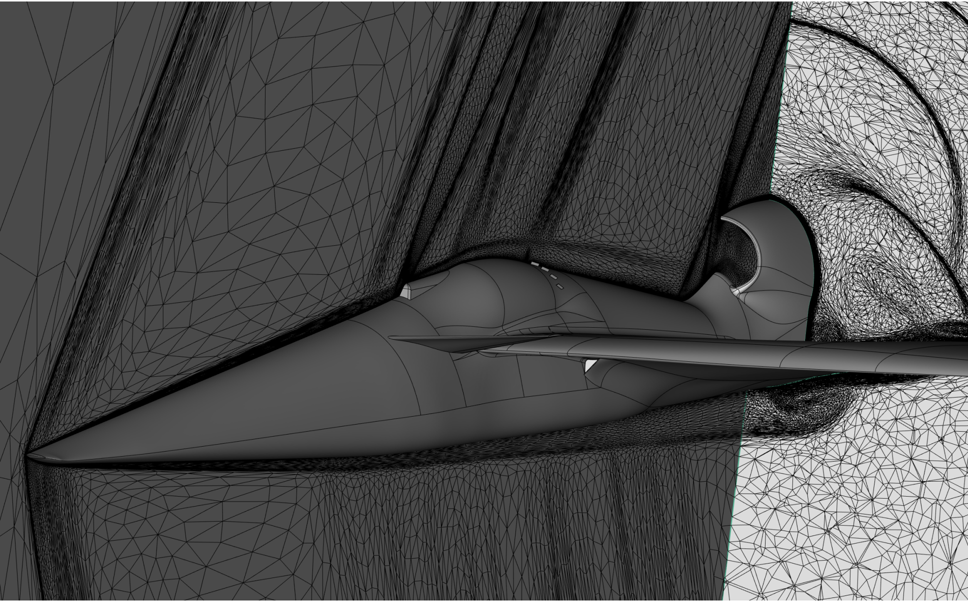

**Improvements over the previous workshops**

- Powered conditions (Inflow/outflow)
- Fully RANS adaptivity
- CAD/Surface adaptivity

**On-going work/Current limitations**

- Distributed parallelization of the flow solver
- Linear system resolution for the adjoint system, currently limited to 10 million vertices then we do not reach convergence
- Adjoint computation on implicit surface (cylinder)