1st AIAA Sonic Boom Prediction Workshop

Sonic boom benchmark cases validation with FASIP

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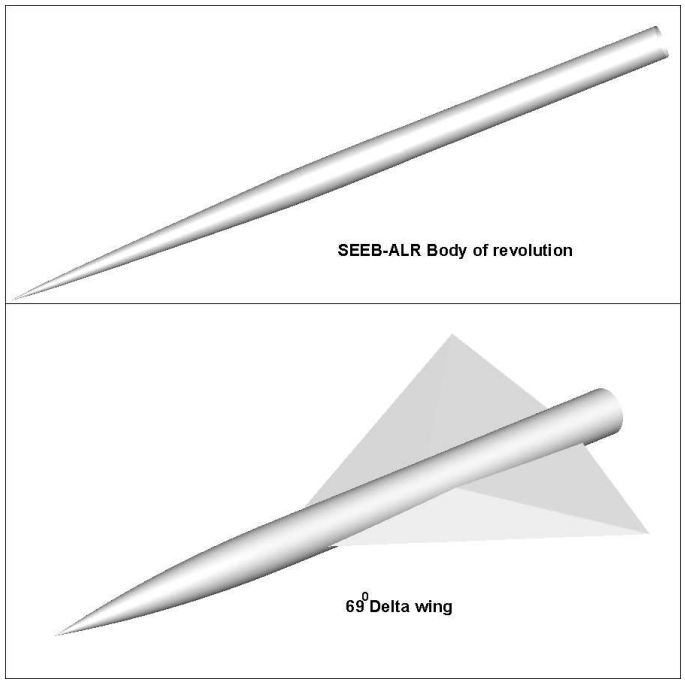
Introduction

- Supersonic airplane requires low sonic boom design
- Sonic boom prediction in the off-body field is challenging
- Assess FASIP code to predict near-field pressure signatures using MUSCL and WENO schemes

Summary of cases analyzed

- SEEB-ALR Body of Revolution
- 69 Delta Wing Body



Numerical Strategies

- 3D Euler equations
- Rielamm solver: Roe's scheme and Low diffusion E-CUSP scheme(Zha etc)
- Reconstruction: 3rd order MUSCL, 3rd and 5th Order WENO scheme
- Time marching: unfactored implicit Gauss-Seidel line relaxation
- High scalability parallel computation

Euler Equations in Generalized Coordinates(ξ, η, ζ)

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{E}}{\partial \xi} + \frac{\partial \mathbf{F}}{\partial \eta} + \frac{\partial \mathbf{G}}{\partial \zeta} = 0$$

where

$$\mathbf{E} = \begin{pmatrix} \bar{\rho} U \\ \bar{\rho}\tilde{u}U + l_x \bar{p} \\ \bar{\rho}\tilde{v}U + l_y \bar{p} \\ \bar{\rho}\tilde{w}U + l_z \bar{p} \\ \left(\bar{\rho}\tilde{e} + \bar{p}\right) U - l_t \bar{p} \\ \bar{\rho}\tilde{\nu}U \end{pmatrix}$$

$$
\begin{aligned}
U &= l_t + \mathbf{l} \bullet \mathbf{V} = l_t + l_x u + l_y v + l_z w \\
V &= m_t + \mathbf{m} \bullet \mathbf{V} = m_t + m_x u + m_y v + m_z w \\
W &= n_t + \mathbf{n} \bullet \mathbf{V} = n_t + n_x u + n_y v + n_z w
\end{aligned}
$$

$$
\mathbf{l} = \frac{\nabla \xi}{J} \mathrm{d}\eta \mathrm{d}\zeta, \ \mathbf{m} = \frac{\nabla \eta}{J} \mathrm{d}\xi \mathrm{d}\zeta, \ \mathbf{n} = \frac{\nabla \zeta}{J} \mathrm{d}\xi \mathrm{d}\eta
$$

$$
l_t = \frac{\xi_t}{J} \mathrm{d}\eta \mathrm{d}\zeta, \ m_t = \frac{\eta_t}{J} \mathrm{d}\xi \mathrm{d}\zeta, \ n_t = \frac{\zeta_t}{J} \mathrm{d}\xi \mathrm{d}\eta
$$

$$
J = \frac{\partial(\xi, \eta, \zeta)}{\partial(x, y, z)} = \frac{1}{x_\xi (y_\eta z_\zeta - y_\zeta z_\eta) - x_\eta (y_\xi z_\zeta - y_\zeta z_\xi) + x_\zeta (y_\xi z_\eta - y_\eta z_\xi)}
$$

The Low Diffusion E-CUSP

• The basic idea of the LDE scheme is to split the inviscid flux into the convective flux E^c and the pressure flux E^p

$$\mathbf{E} = E^c + E^p = \begin{pmatrix} \rho U \\ \rho u U \\ \rho v U \\ \rho w U \\ \rho e U \end{pmatrix} + \begin{pmatrix} 0 \\ l_x p \\ l_y p \\ l_z p \\ p\overline{U} \end{pmatrix}$$

• Ability to capture crisp shock and contact discontinuities
• Simpler and more CPU efficient than Roe scheme

The 5th Order WENO Scheme*

• The interface flux, $E_{i+\frac{1}{2}} = E(Q_L, Q_R)$, is evaluated by determining the conservative variables Q_L and Q_R using fifth-order WENO scheme.

$$\left(Q_{i+\frac{1}{2}}\right)_L = \omega_0 q_0 + \omega_1 q_1 + \omega_2 q_2$$

where

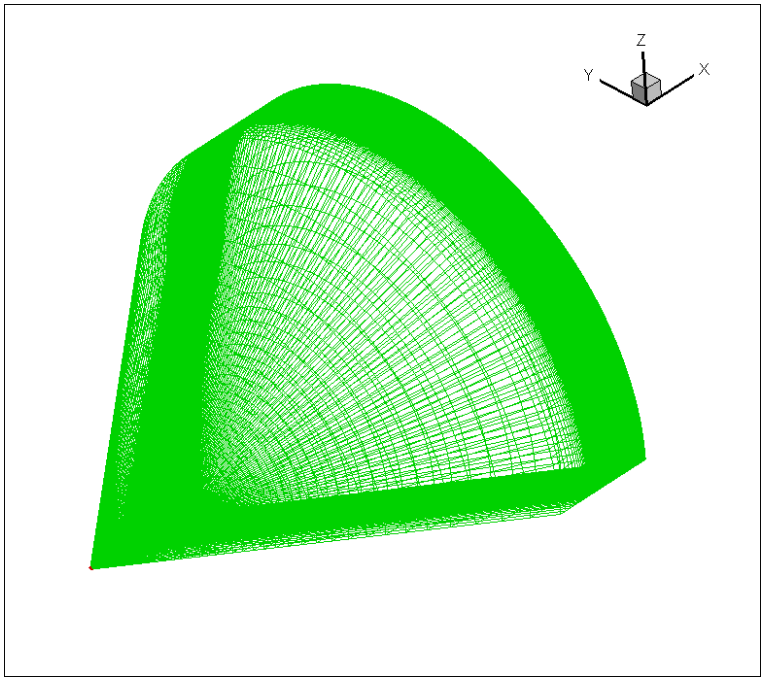
$$\begin{aligned} q_0 &= \tfrac{1}{3}Q_{i-2} - \tfrac{7}{6}Q_{i-1} + \tfrac{11}{6}Q_i \\ q_1 &= -\tfrac{1}{6}Q_{i-1} + \tfrac{5}{6}Q_i + \tfrac{1}{3}Q_{i+1} \\ q_2 &= \tfrac{1}{3}Q_i + \tfrac{5}{6}Q_{i+1} - \tfrac{1}{6}Q_{i+2} \end{aligned}$$

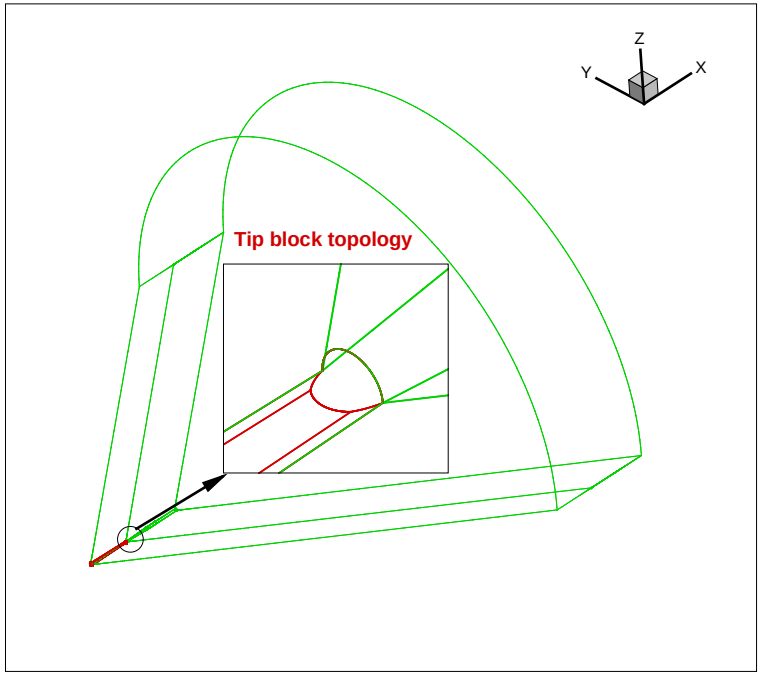
Shen et al, AIAA Journal, vol 47, pp.331-344, 2009

$$\alpha_k = \frac{C_k}{\epsilon + IS_k},\ k = 0, \ldots, r-1$$
$$C_0 = 0.1,\ C_1 = 0.6,\ C_2 = 0.3$$
$$IS_0 = \frac{13}{12}\left(Q_{i-2} - 2Q_{i-1} + Q_i\right)^2 + \frac{1}{4}\left(Q_{i-2} - 4Q_{i-1} + 3Q_i\right)^2$$
$$IS_1 = \frac{13}{12}\left(Q_{i-1} - 2Q_i + Q_{i+1}\right)^2 + \frac{1}{4}\left(Q_{i-1} - 4Q_i + 3Q_{i+1}\right)^2$$
$$IS_2 = \frac{13}{12}\left(Q_i - 2Q_{i+1} + Q_{i+2}\right)^2 + \frac{1}{4}\left(Q_i - 4Q_{i+1} + 3Q_{i+2}\right)^2$$

$\epsilon = 10^{-6} \longrightarrow \epsilon = 10^{-2}$, which significantly improves the convergence rate and level for the transonic flow with shock waves. If no shock,$\epsilon = 0.3$ is recommended to use.

SEEB-ALR Body of Revolution



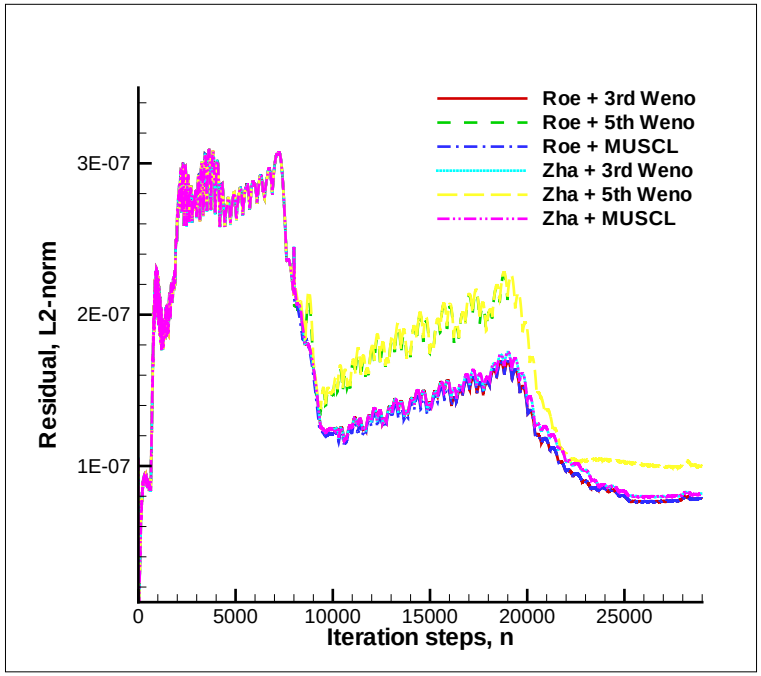


Structured grid for Euler solver; Grid alignment with mach angle

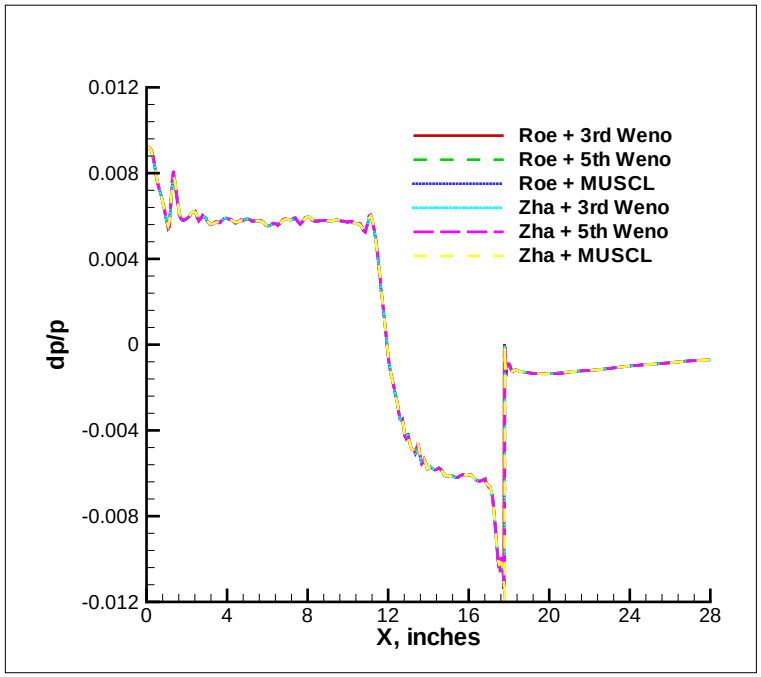
Coarse mesh size: 65*97*353; Refined mesh size: 97*129*593

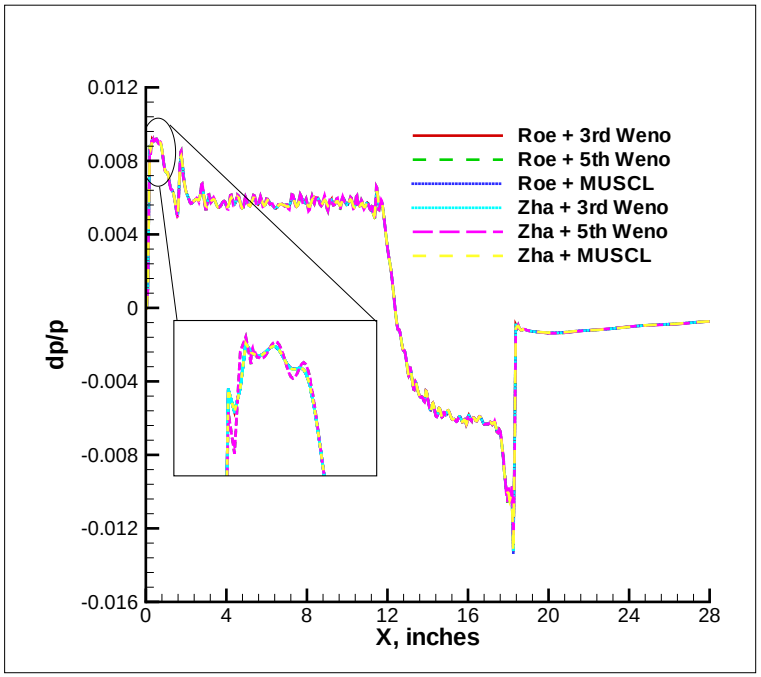
SEEB-ALF: Flow Solver convergence

- Iterations: 16000
- Convergence criteria: L2-norm
- Convergence history:



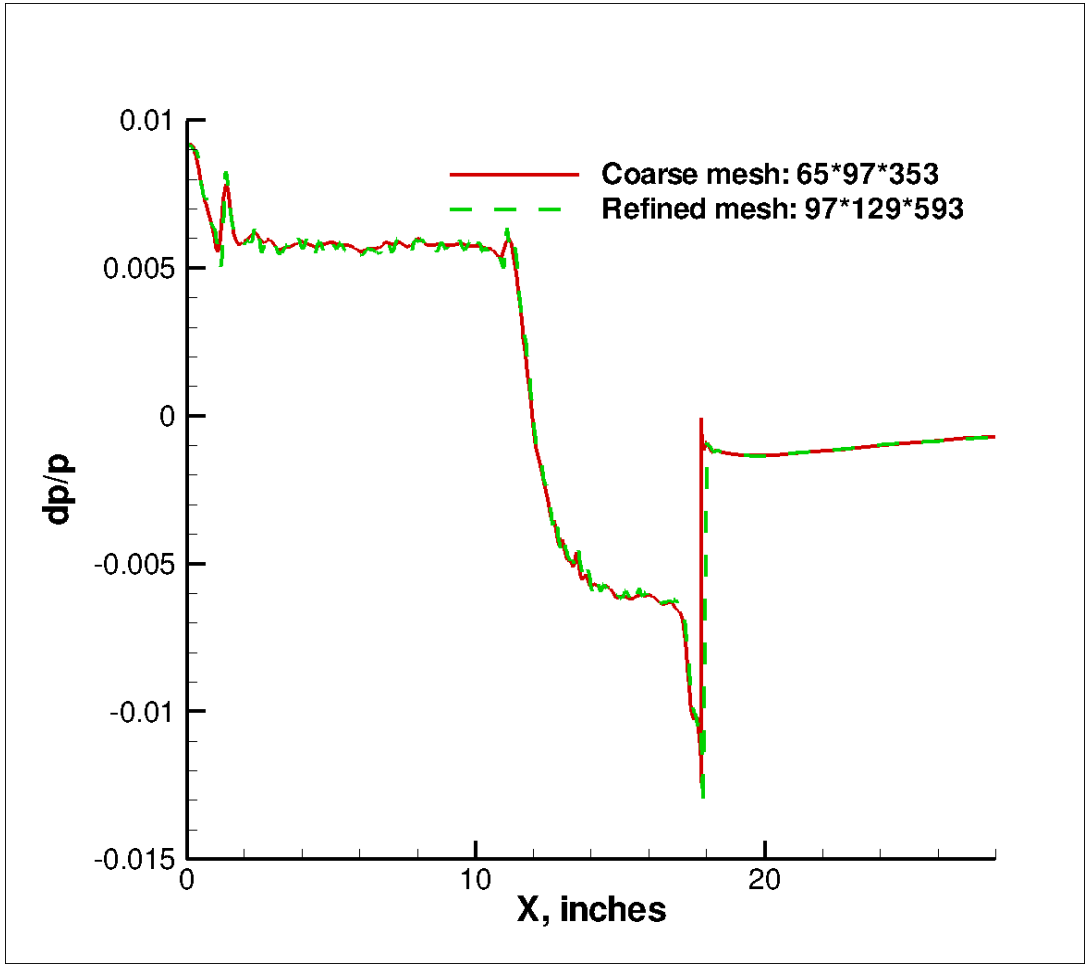
SEEB-ALR *





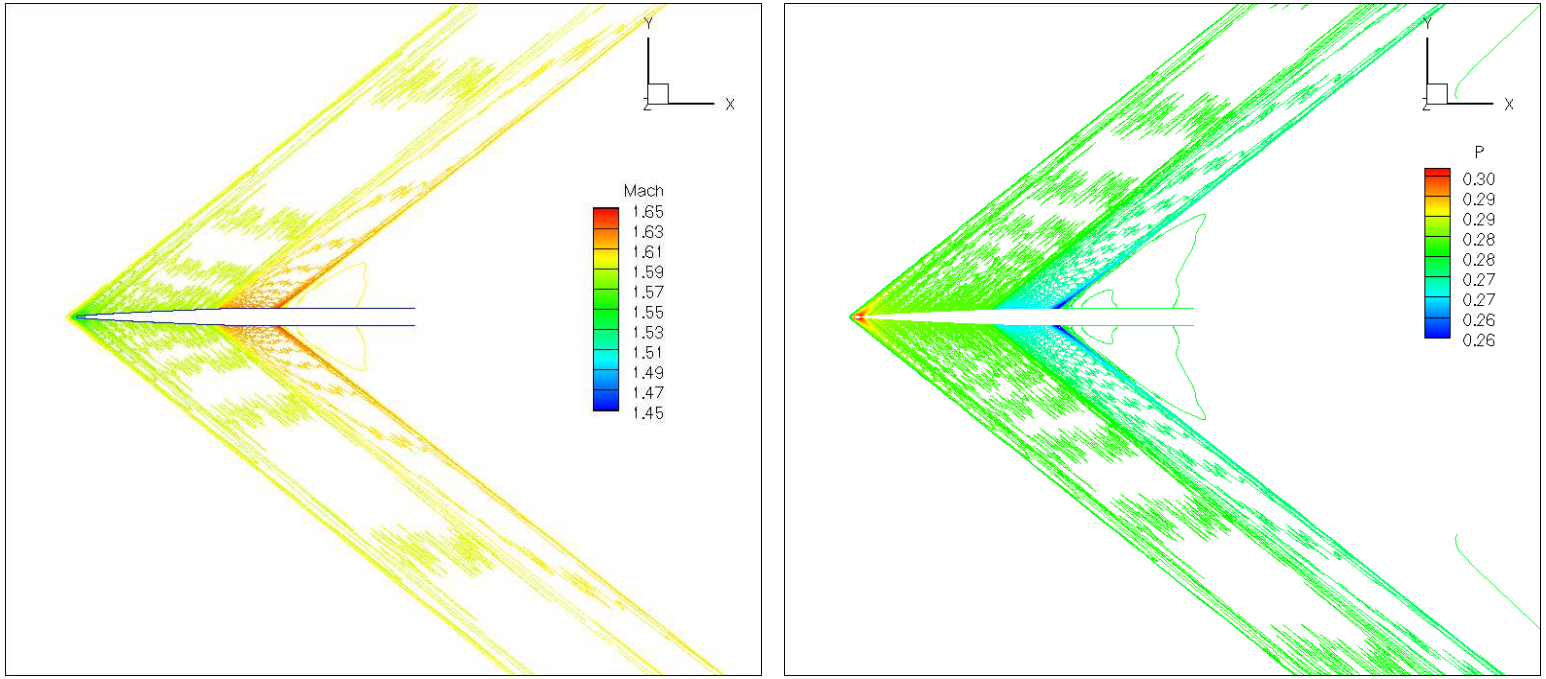
*Extracted near field signatures with different schemes, left: coarse mesh; right: refined mesh

SEEB-ALR *



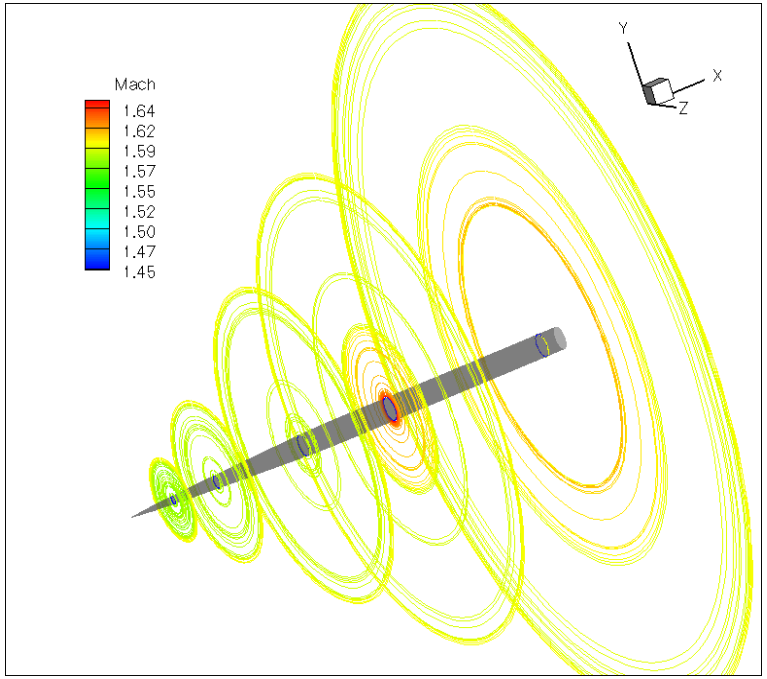
*Mesh resolution comparisions with the 3rd-Weno schemes

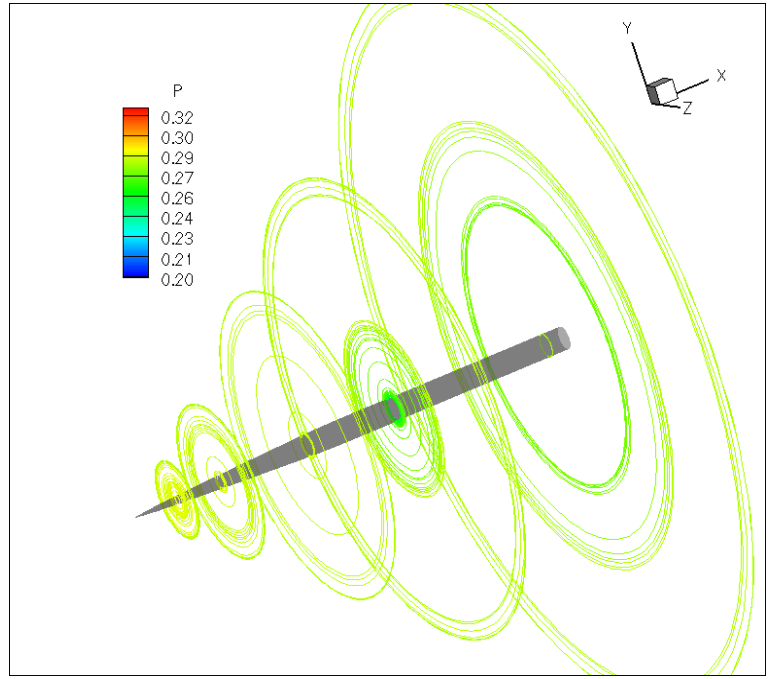
SEEB-ALR *



*Countour plots, left: Mach line; right: pressure line

SEEB-ALR *





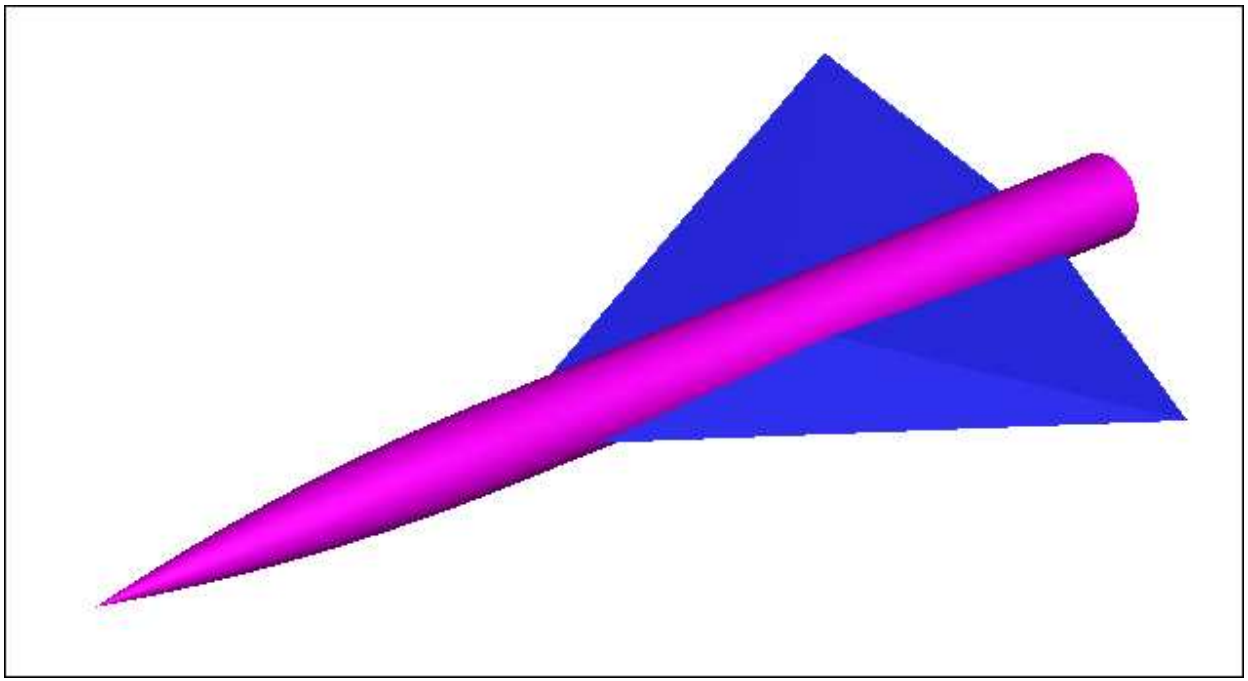
*Countour plots, left: Mach line; right: pressure line

69° Delta wing • M=1.7, Gama=1.4

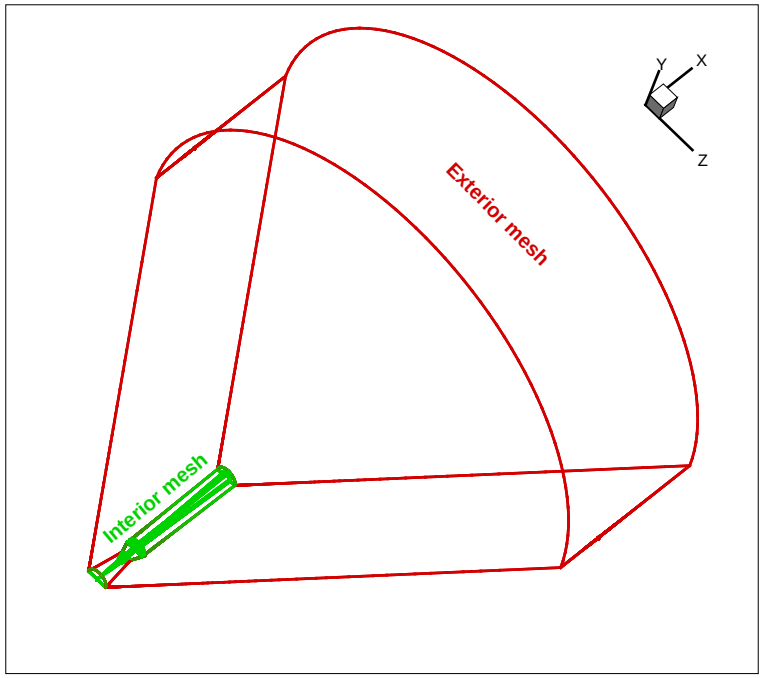
• Angle of Attack(AoA): 0.0, 2.079, 3.588, 4.74

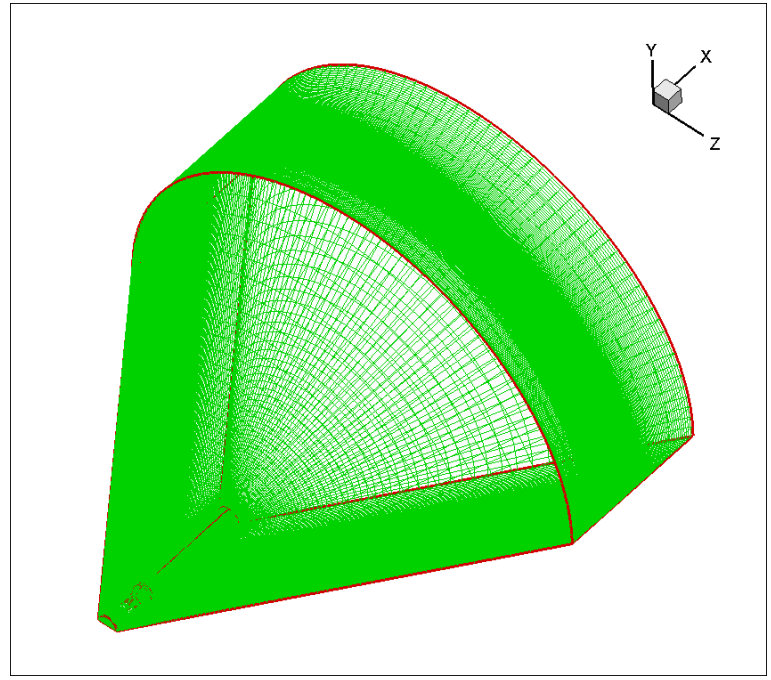
• Extracted near field at H=21.2, 24.8, 31.8 inches body lengths below the model

• Geometry:



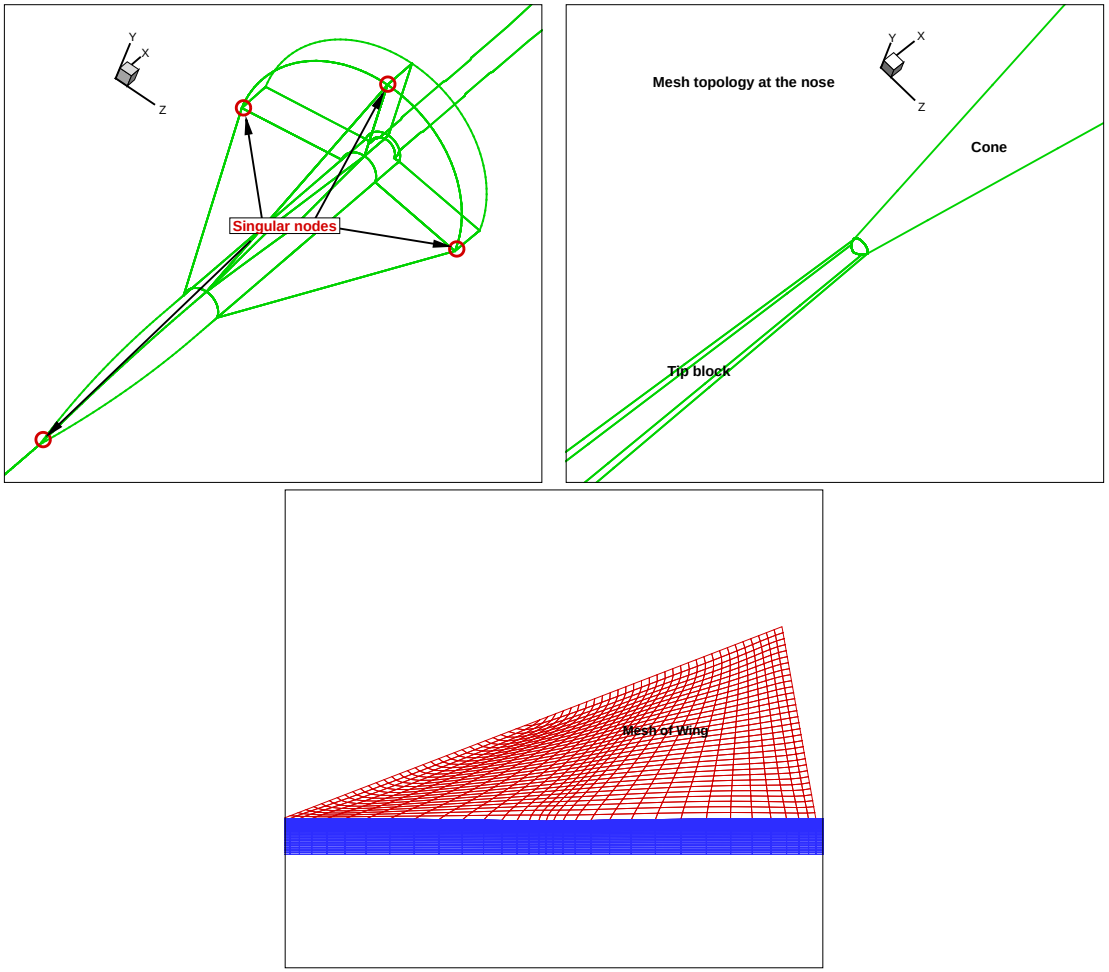
Mesh of Delta wing *





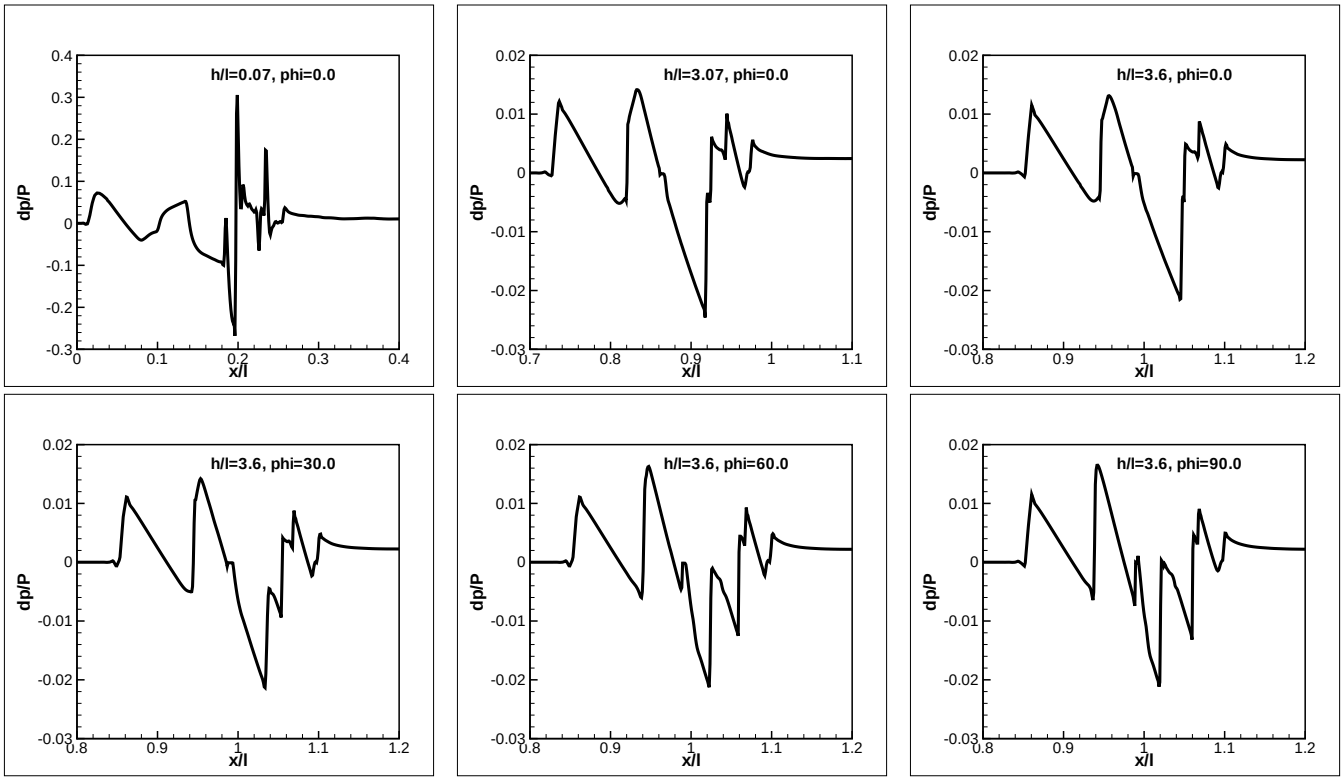
*The grids near the body are regenerated. The external grids are the same as that of provoided by the workshop

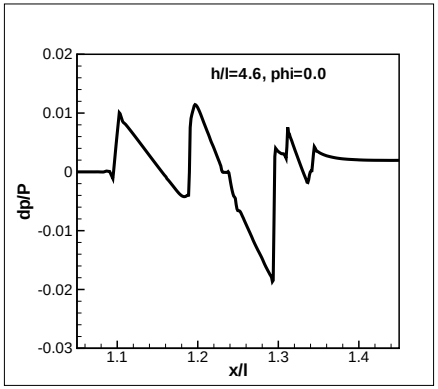
Mesh of Delta wing *

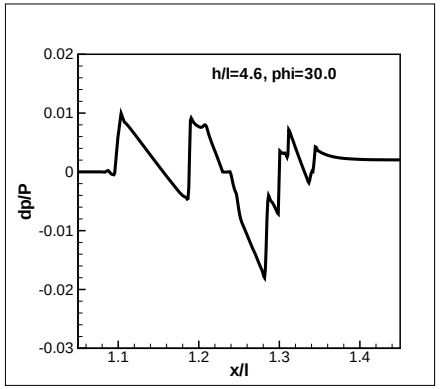


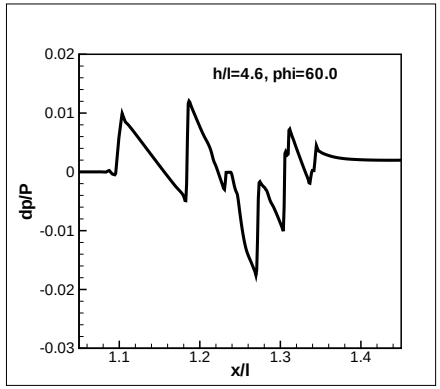
*Mesh regeneration: remove the singular node

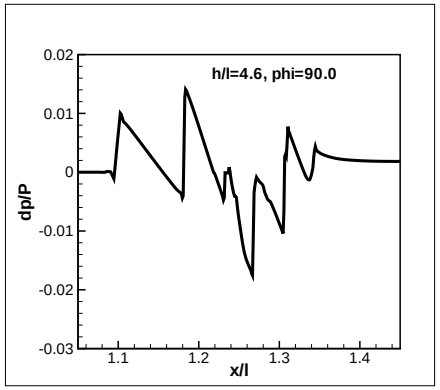
Extracted signatures: AoA=0.0, to the workshop



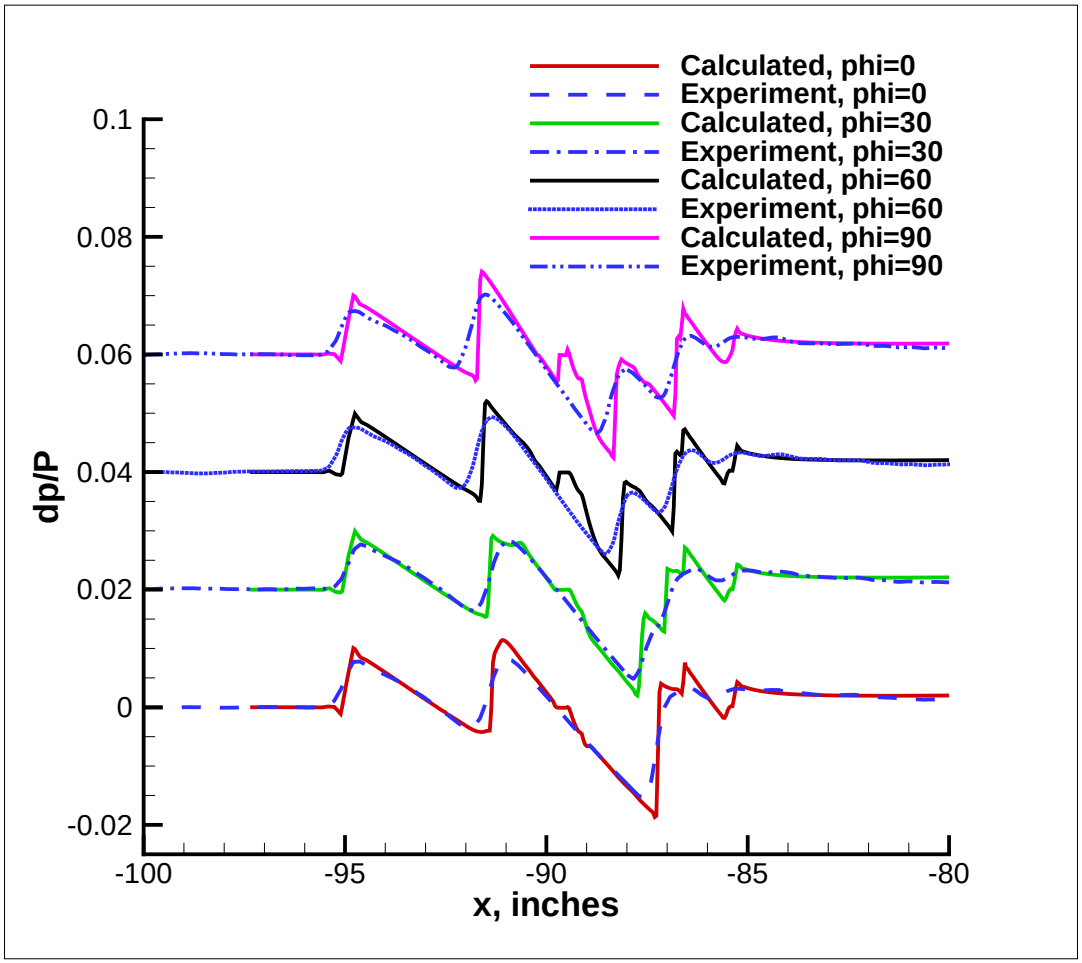

h/l=4.6, phi=0.0
dp/P
x/l
0.02
0.01
0
-0.01
-0.02
-0.03
1.1
1.2
1.3
1.4


h/l=4.6, phi=30.0
dp/P
x/l
0.02
0.01
0
-0.01
-0.02
-0.03
1.1
1.2
1.3
1.4


h/l=4.6, phi=60.0
dp/P
x/l
0.02
0.01
0
-0.01
-0.02
-0.03
1.1
1.2
1.3
1.4

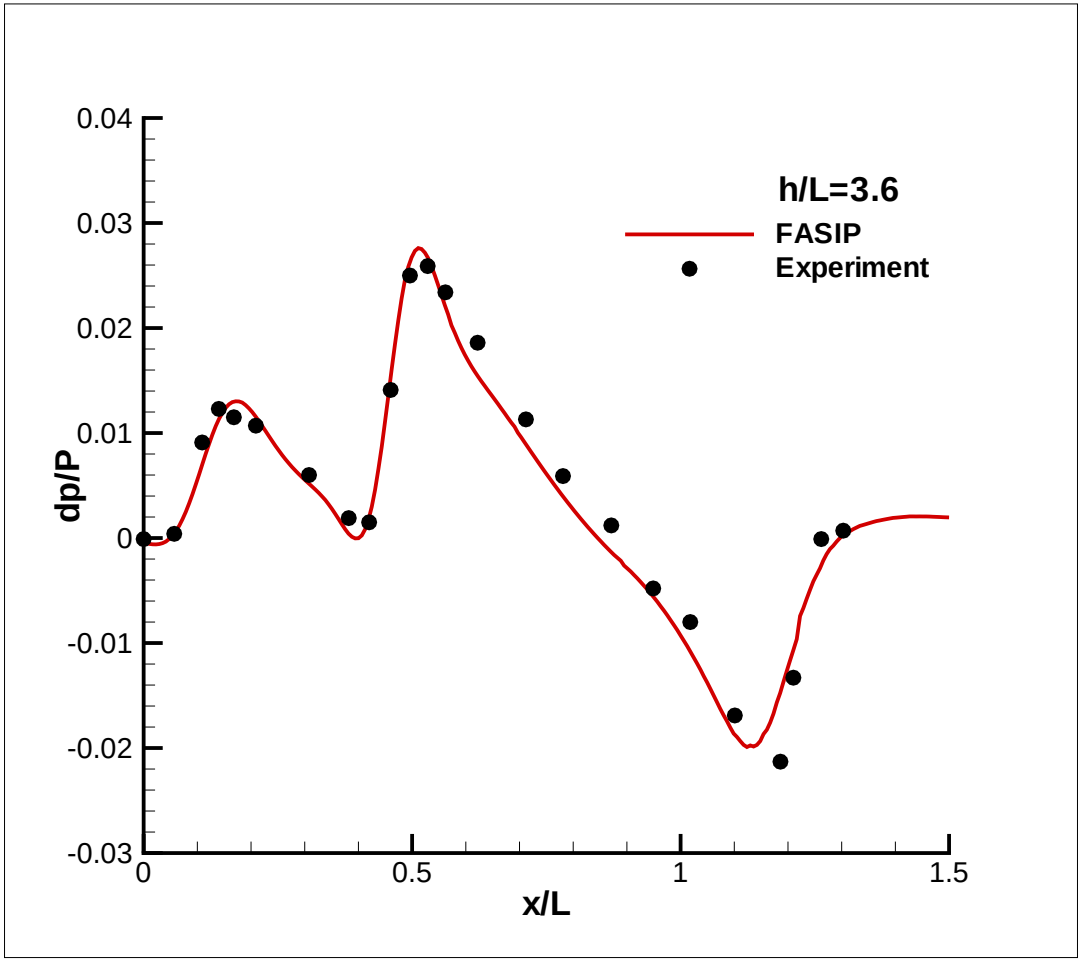

h/l=4.6, phi=90.0
dp/P
x/l
0.02
0.01
0
-0.01
-0.02
-0.03
1.1
1.2
1.3
1.4

Extracted signatures: AoA=0.0, h=31.6 *

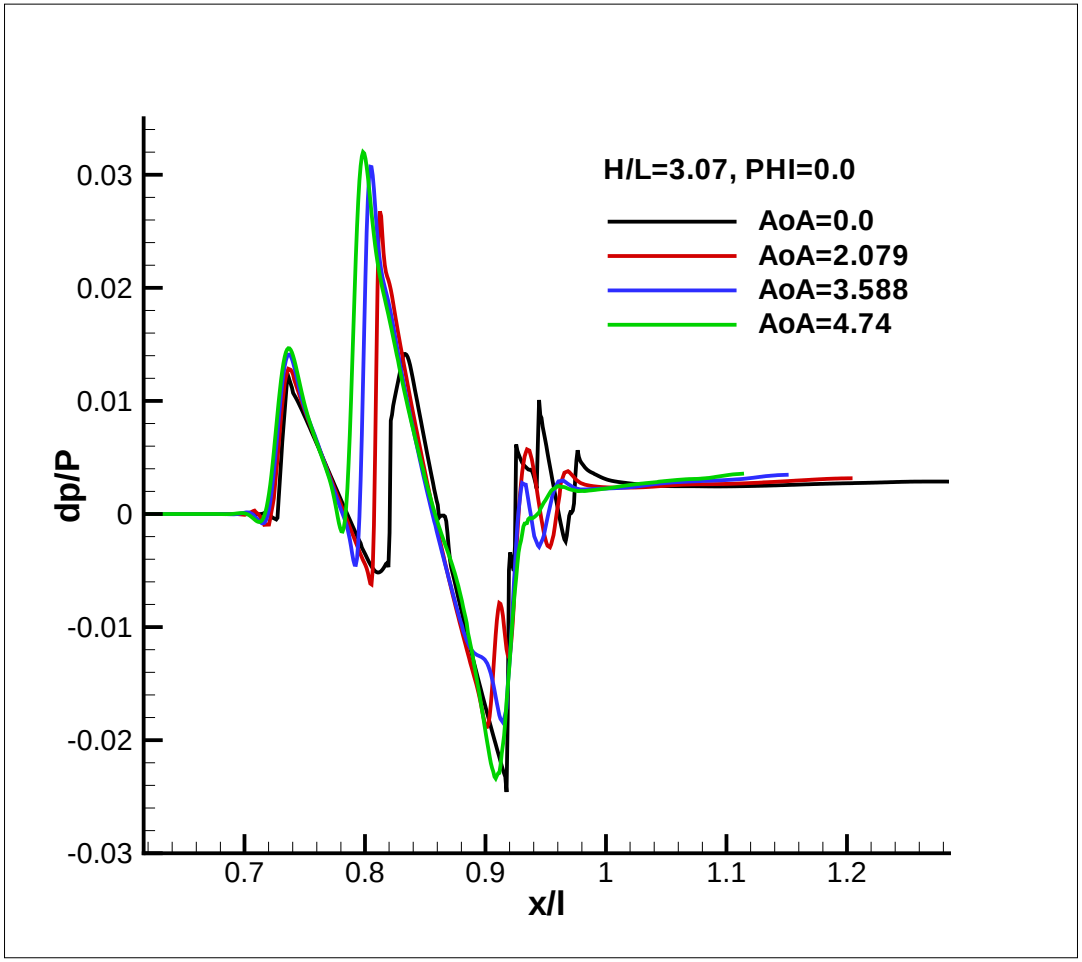


*Curves are offset by 0.02 for each signal with phi larger than 0.0

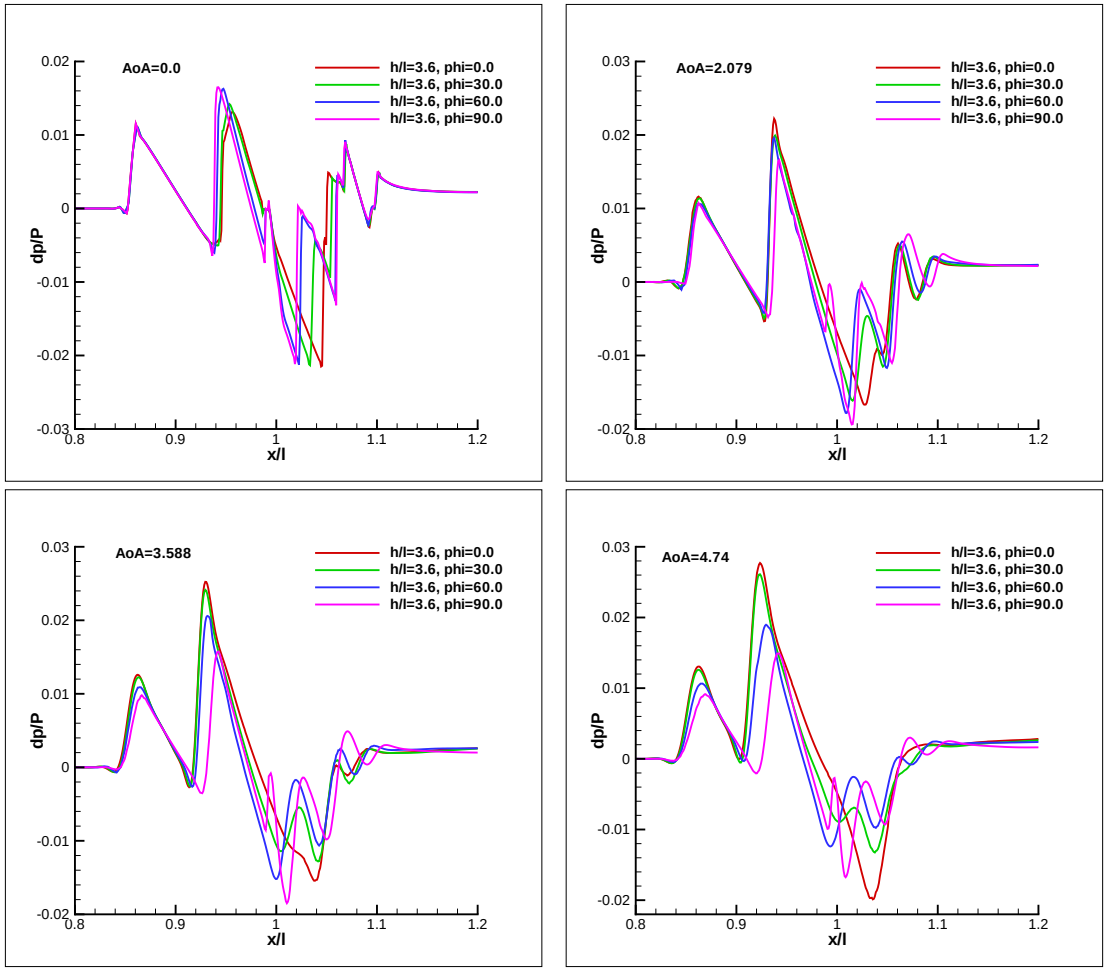
Extracted signatures: AoA=4.74, h/l=3.6



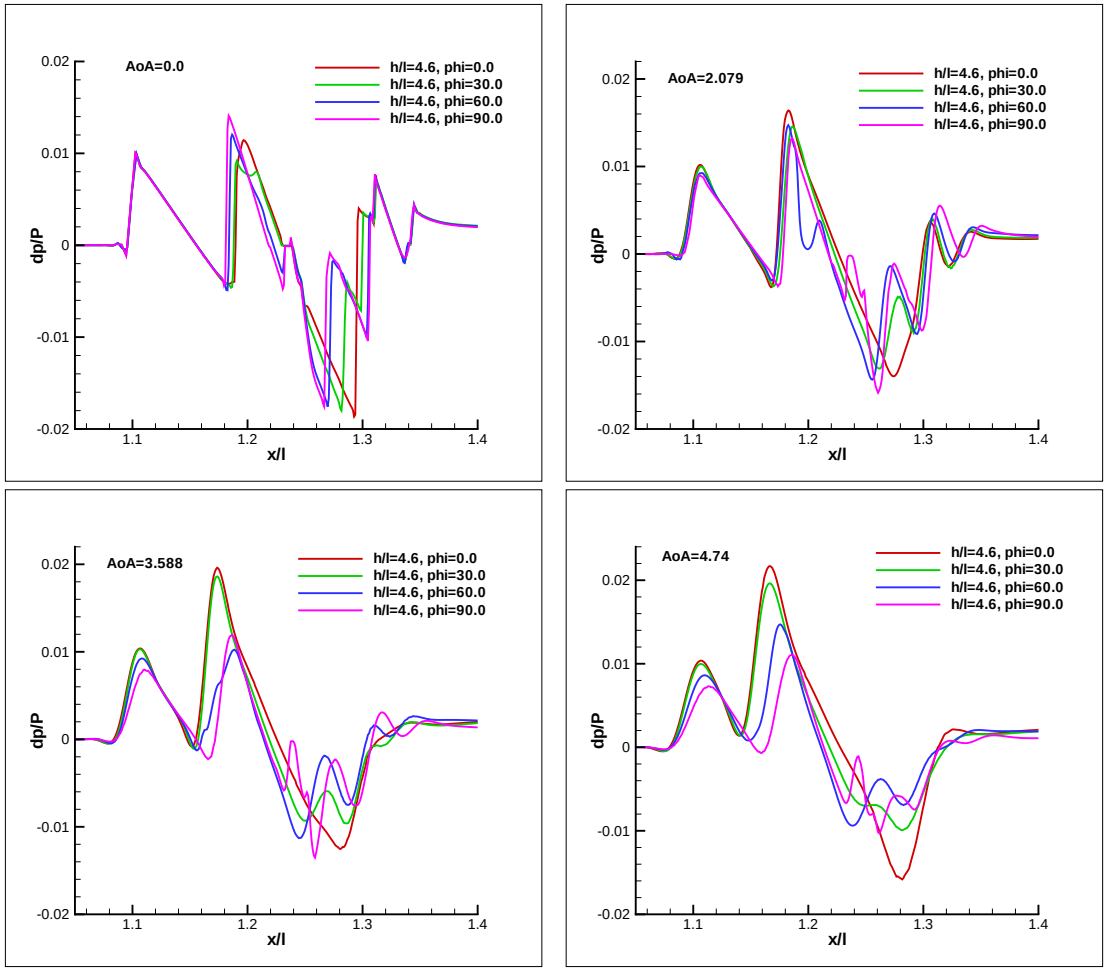
Extracted signature comparisons with different AoA at h/l=3.07



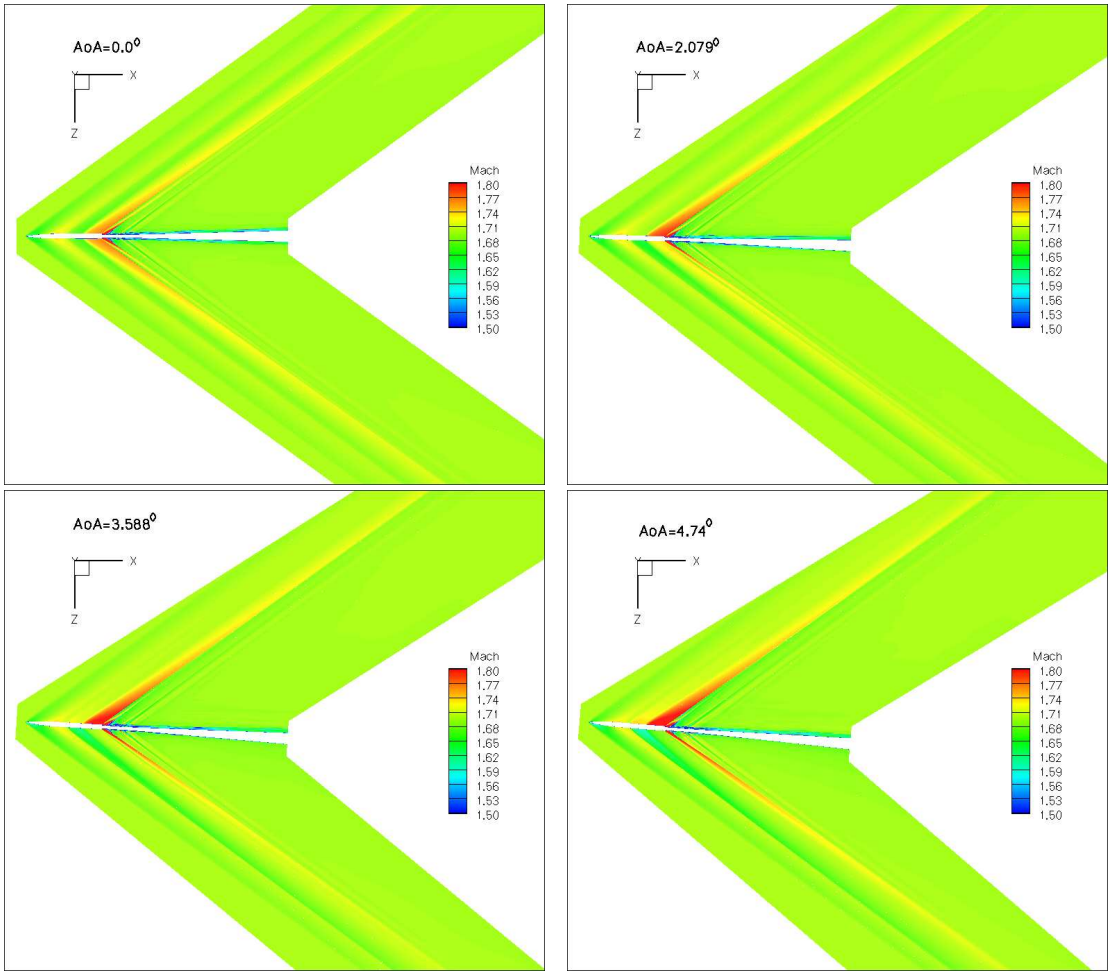
Extracted signature comparisons with different AoA at h/l=3.6



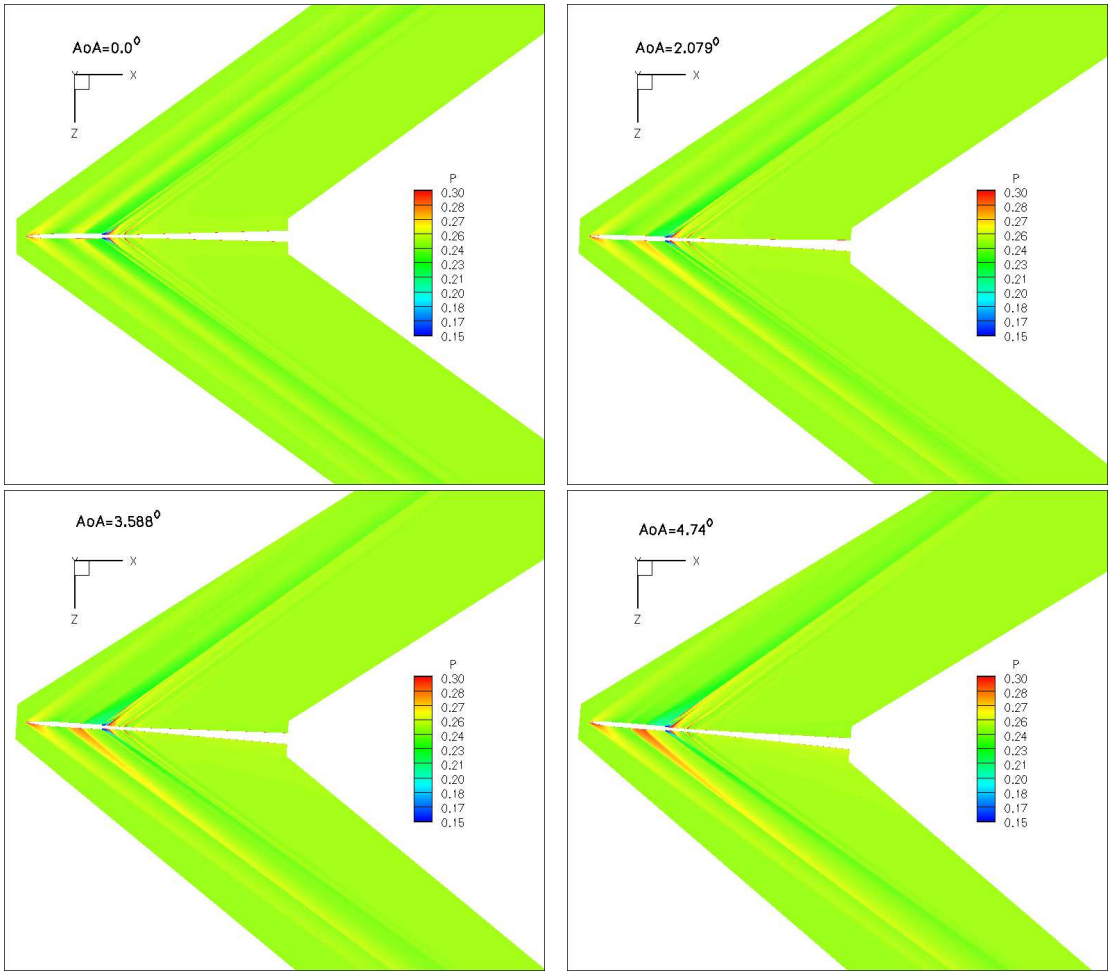
Extracted signature comparisons with different AoA at h/l=4.6



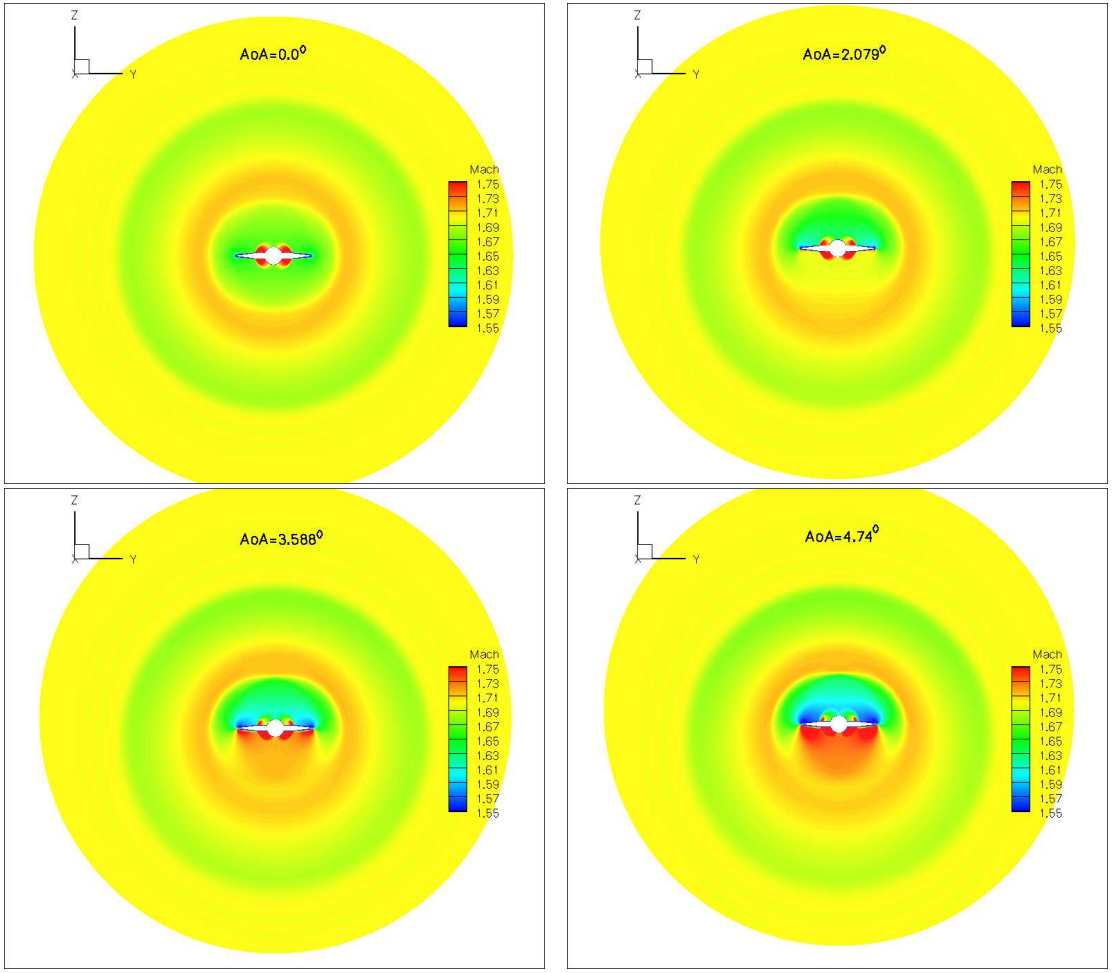
Mach contours varied with AoA in streamwise



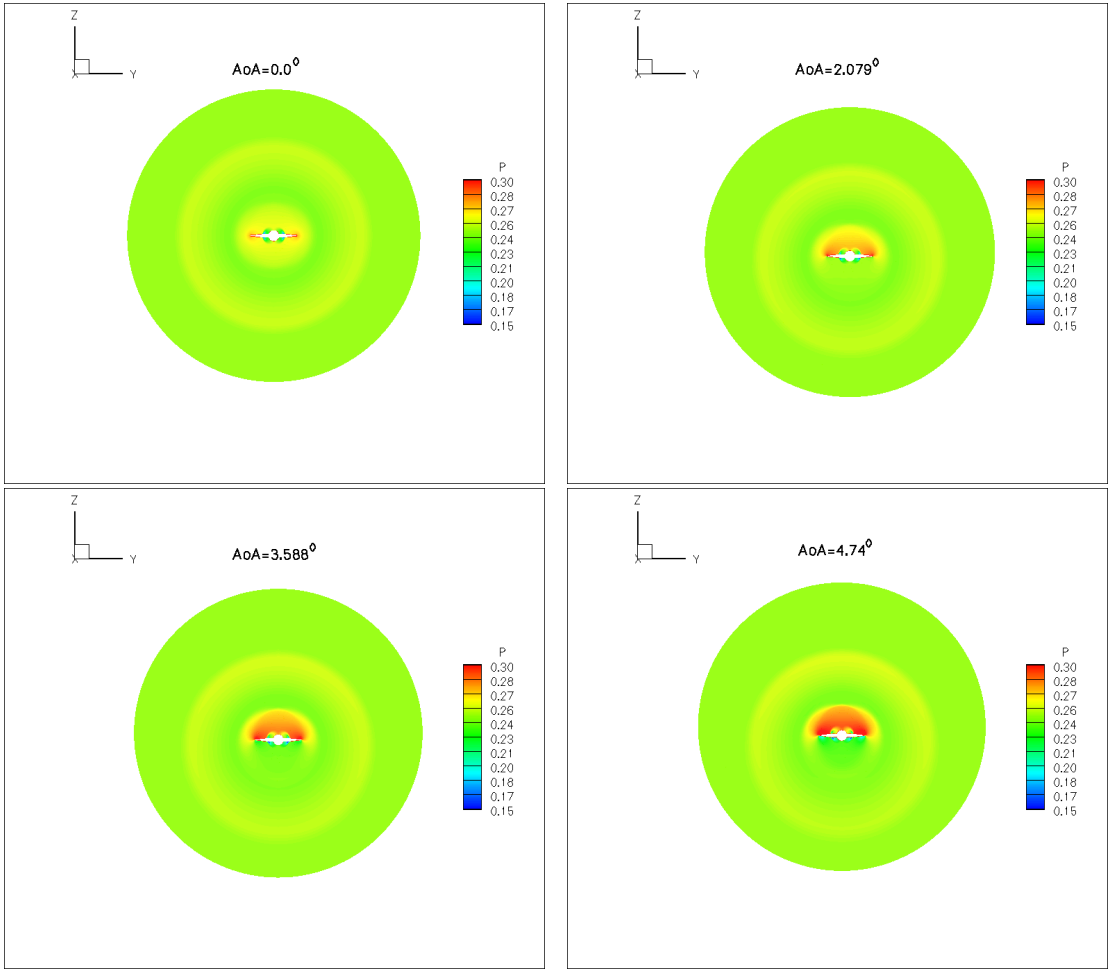
Pressure contours varied with AoA in streamwise



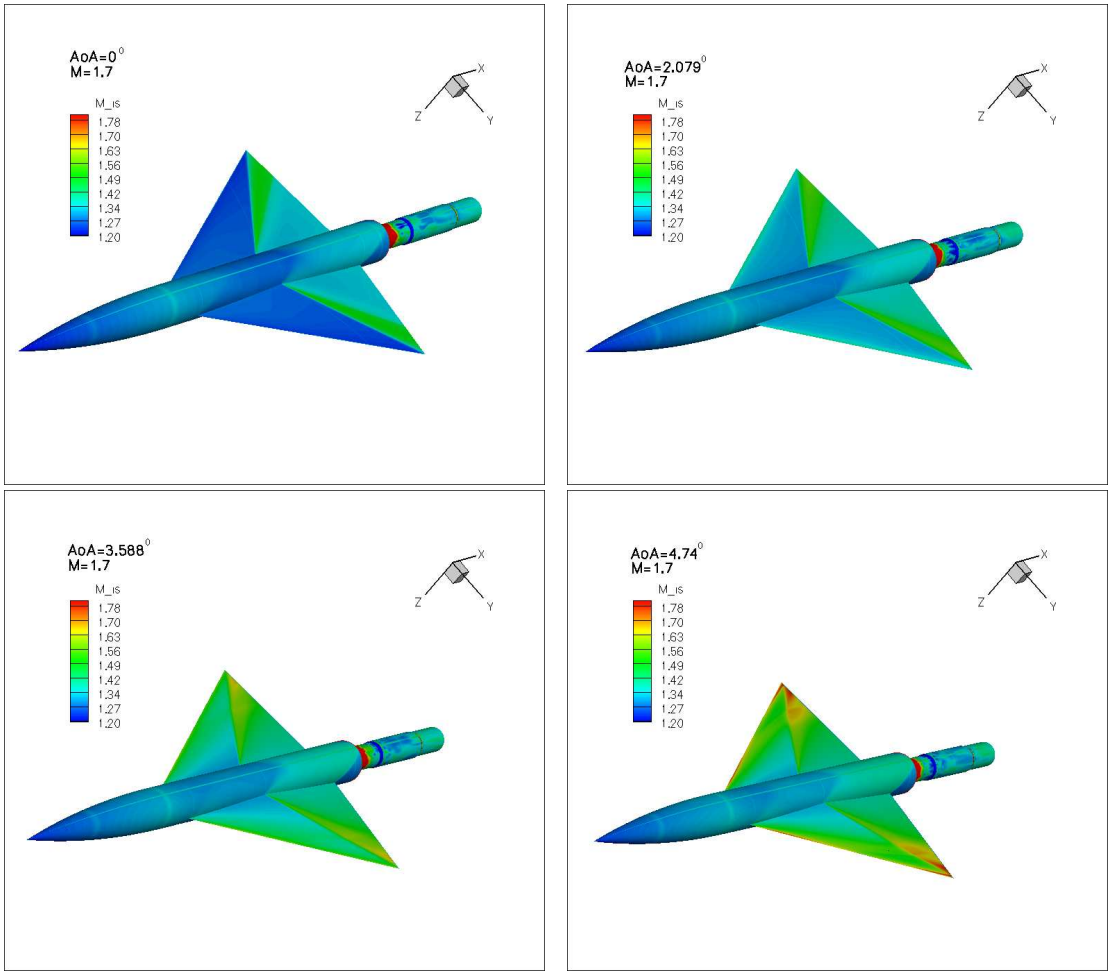
Mach contours varied with AoA cross the wing



Pressure contours varied with AoA cross the wing



Isentropic mach number distribution around the body surface



Conclusions

• 3rd order MUSCL, 3rd order WENO and 5th order schemes give the same results for the SEEB-ALR BoR

•69 Delta Wing use 3rd order MUSCL

•Results appear to be reasonable