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Adjoint Error Estimation During Prediction of Sonic Booms

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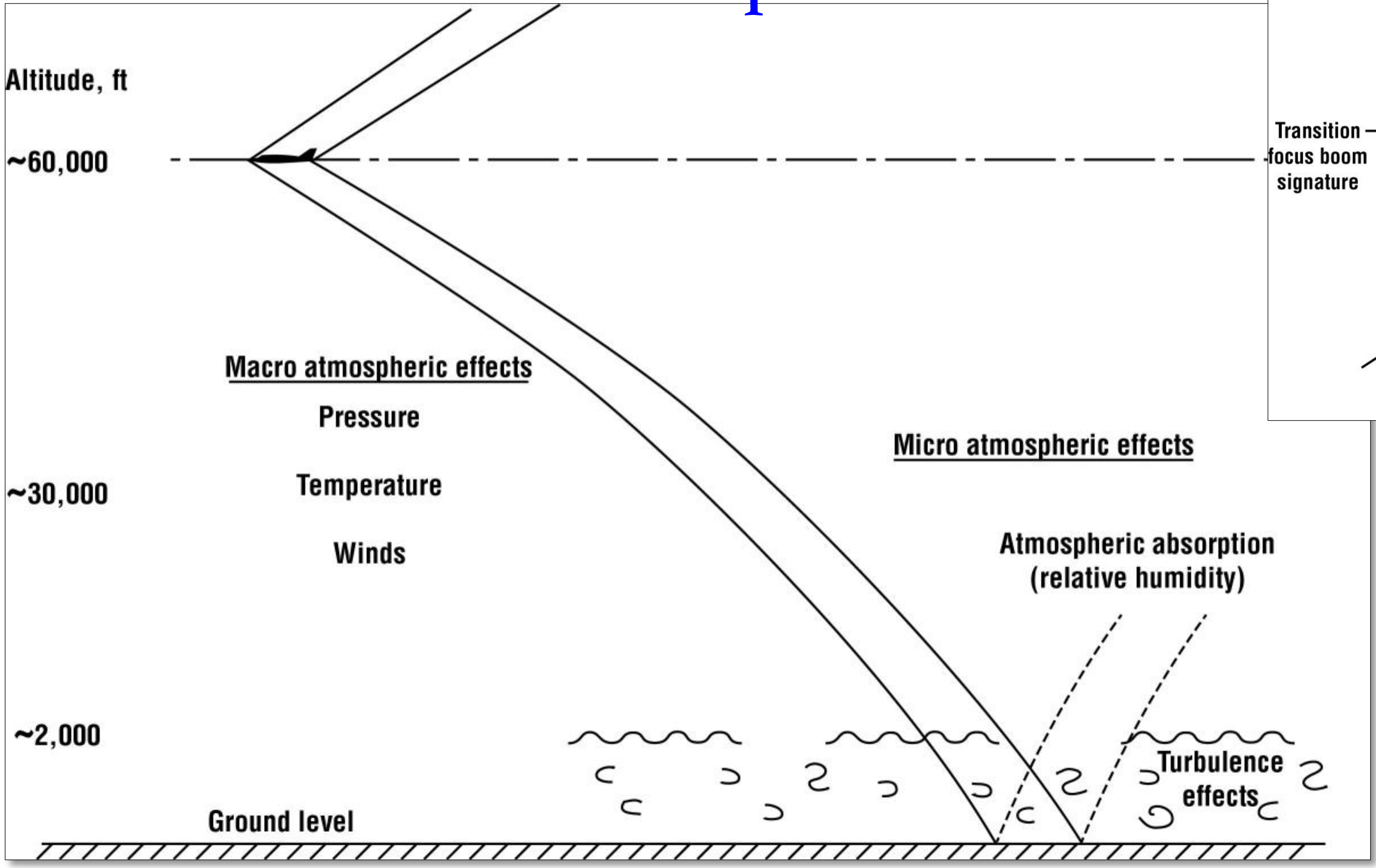
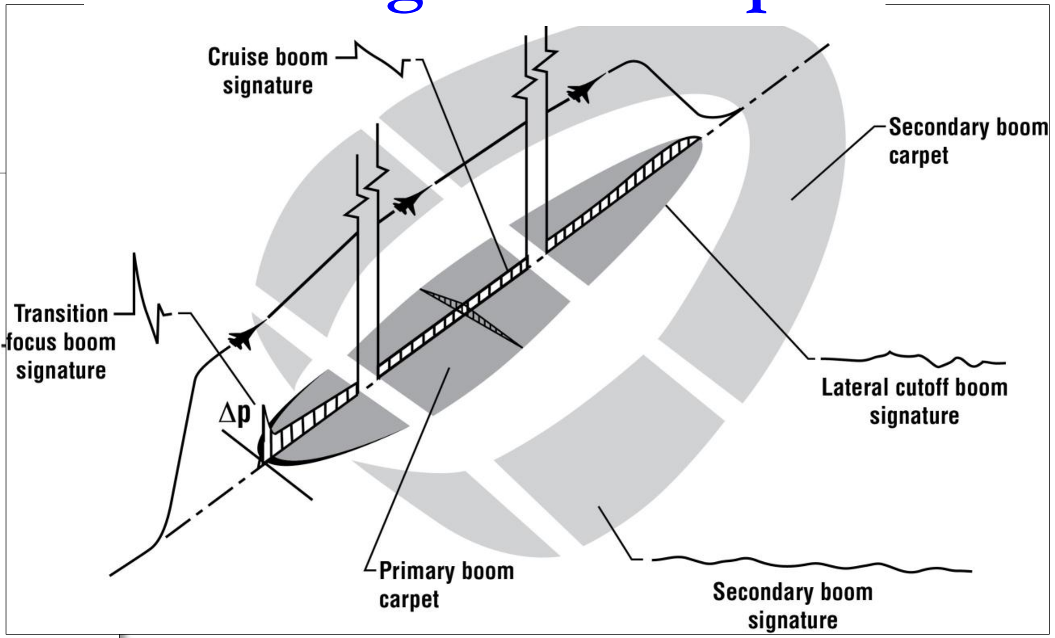
A Storied Legacy, A Soaring Future

Sonic Boom Primer

Sonic boom with atmospheric effects



Boom signature carpet



Introduction: Sonic Boom Modeling

sBOOM

- Propagation based on lossy Quasi-1D Burgers equation

$$\frac{\partial P}{\partial \sigma} = P\frac{\partial P}{\partial \tau} + \frac{1}{\Gamma}\frac{\partial^2 P}{\partial \tau^2} + \sum_{\nu=1}^{2} C_\nu \frac{\partial^2}{1+\theta_\nu \frac{\partial}{\partial \tau}} P - \frac{\frac{\partial A}{\partial \sigma}}{2A} P + \frac{\frac{\partial(\rho_0 c_0)}{\partial \sigma}}{2\rho_0 c_0} P$$

- **Featur**
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    signa
  - Hori
  - Acce
    climl
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    - *Unique*

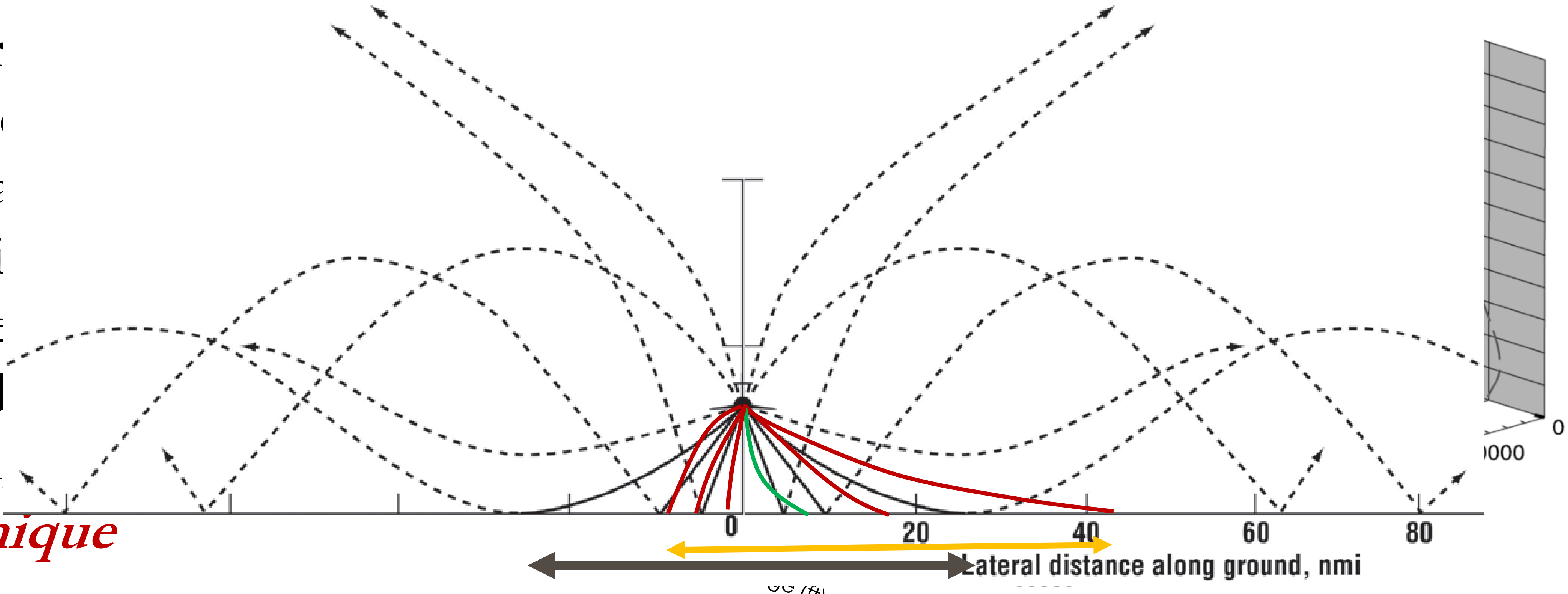


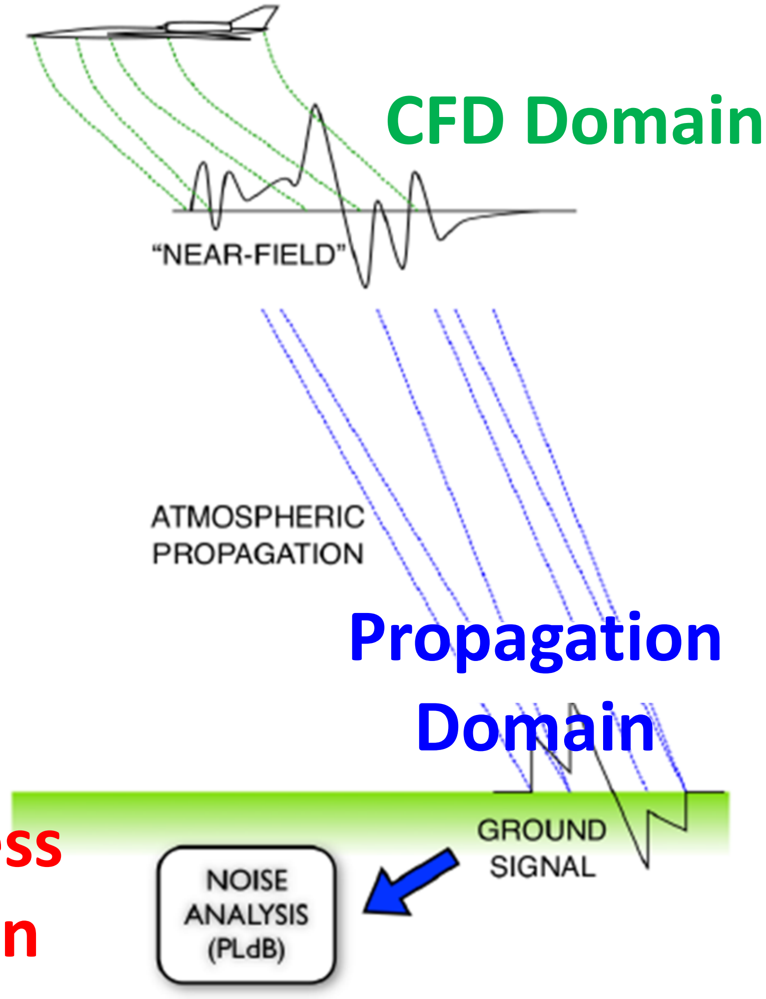
Figure source: "sBOOM Propagation for the Second AIAA Sonic Boom Prediction Workshop" Aftosmis, M.J., Anderson, G. R., and Nemec, M., 2nd AIAA Sonic Boom Prediction Workshop, Jan 2017, Grapevine TX

Sonic Boom Calculation Overview

CFD Domain

Propagation Domain

Loudness Domain



Wintzer AIAA 2015-2260

Cart3D
FUN3D
USM3D
LAVA …

Typically extract pressure data on a cylinder 3BL in radius

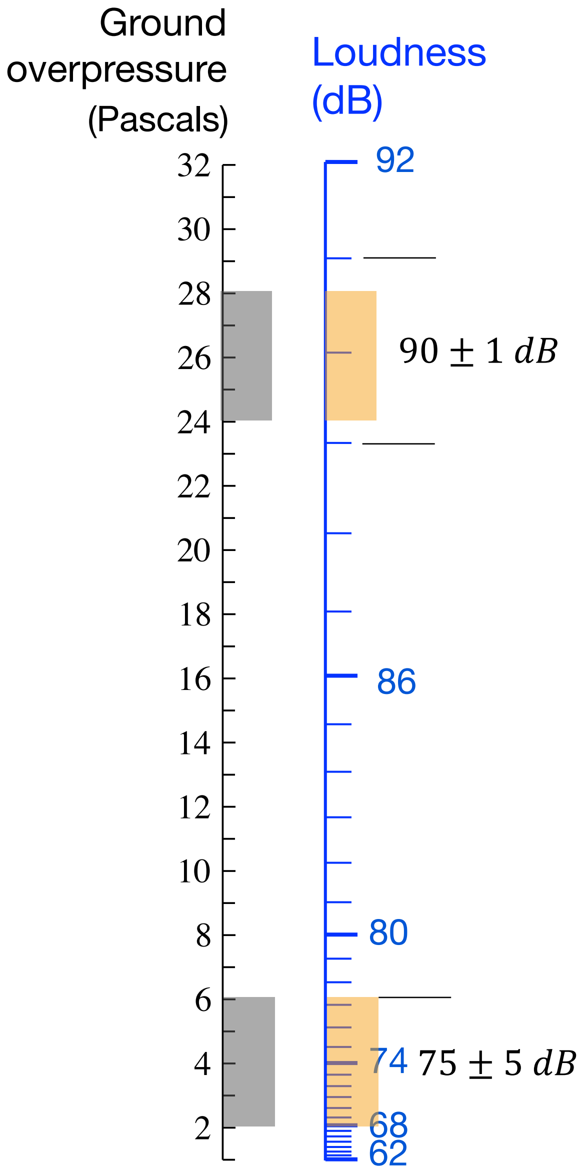
PCBoom
sBOOM
LMBOOM
GACBoom…

Includes atmospheric absorption, winds, turbulence, etc

LCASB

Loudness metrics including PLdB, ASEL, etc

Sonic Boom Modeling: Numerical Challenges

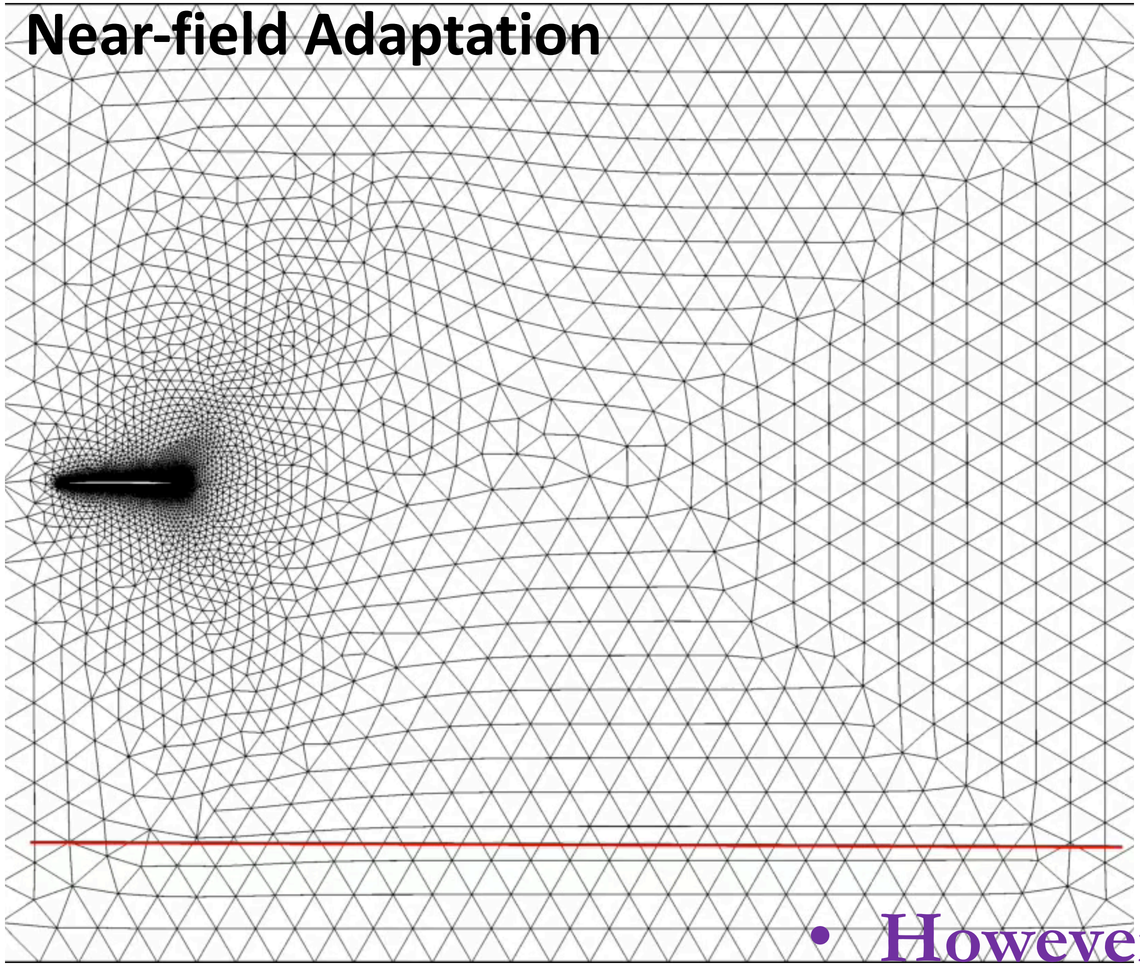


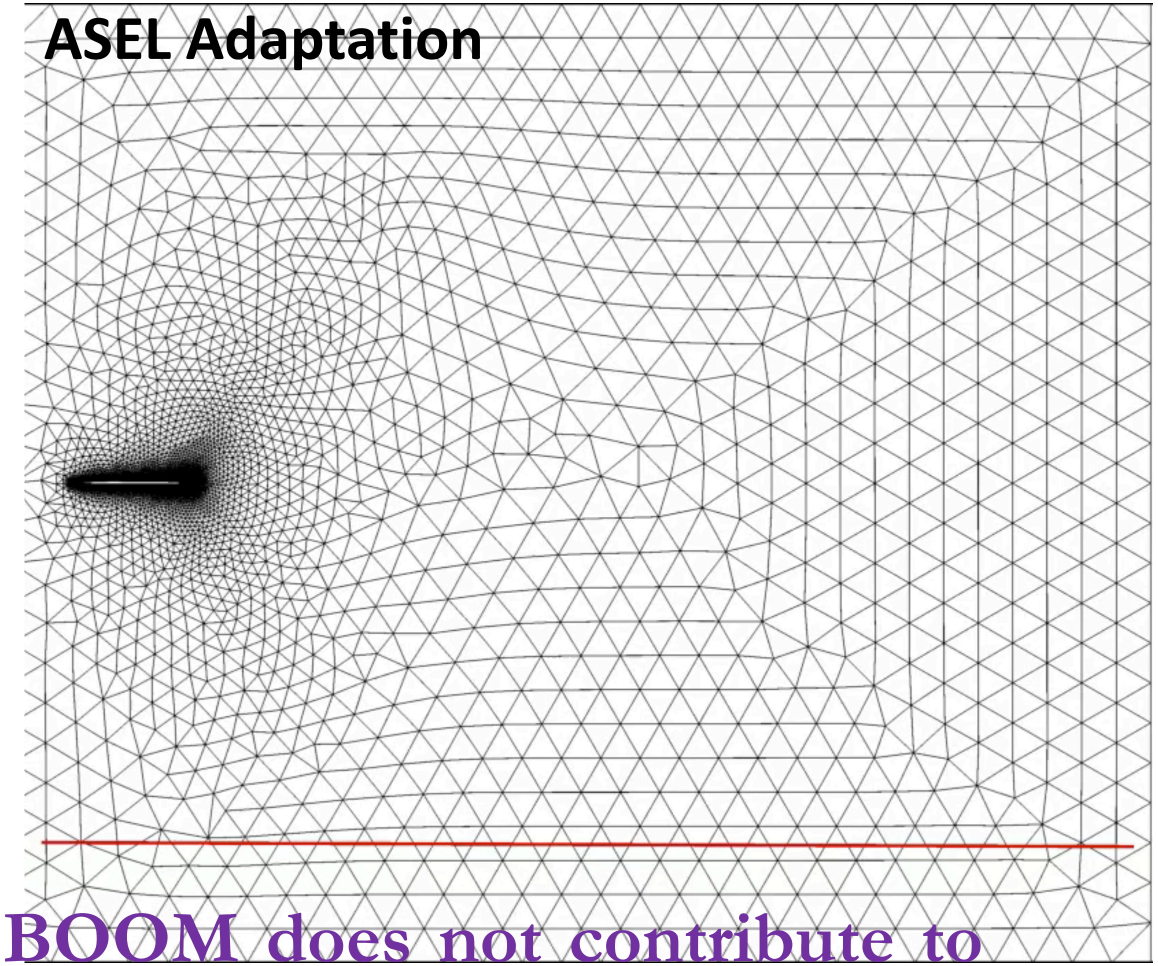
- Impact of an error of $\pm$2 Pa on a 90 PLdB signal may be less than 1 dB
- The same error on a 75 PLdB signal could be $\pm$ 5 dB
- **Mesh requirements and analysis uncertainty increase as aircraft designs becomes quieter!**

Idea Source: "sBOOM Propagation for the Second AIAA Sonic Boom Prediction Workshop" Aftosmis, M.J., Anderson, G. R., and Nemec, M., 2nd AIAA Sonic Boom Prediction Workshop, Jan 2017, Grapevine TX

Motivation

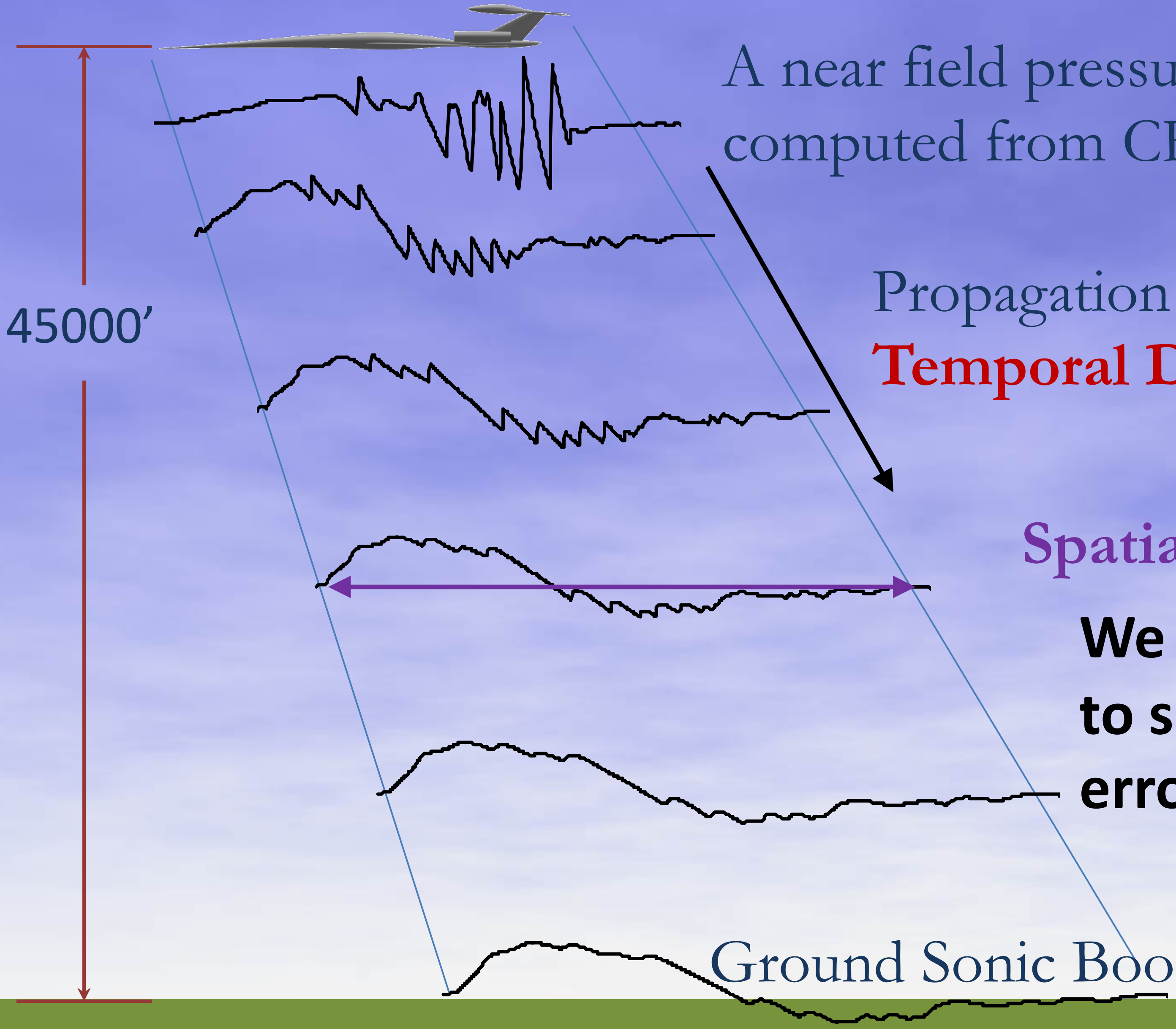
- Recent CFD developments include output-based mesh adaptation to resolve near-field as well as sonic boom loudness (ASEL)





- However sBOOM does not contribute to the error to drive adaptation

A near field pressure distribution computed from CFD



Propagation toward ground:
Temporal Discretization

Spatial Discretization

We are restricting ourselves to spatial discretization errors only in this talk

Ground Sonic Boom Signature

Sonic Boom Adjoints

- Sonic boom numerical modeling:

k_n	Blokhintzev scaling term
A^n, B^n	N_2 Relaxation matrices
$A_{n,2}, B_{n,2}$	O_2 Relaxation matrices
$A_{n,3}, B_{n,3}$	Absorption matrices
$f(t_n)$	Nonlinear terms

sBOOM:

$$A_n q_n = k_n B_n p_{n-1}$$

$$A_{n,2} r_n = B_{n,2} q_n$$

$$A_{n,3} t_n = B_{n,3} r_n$$

$$p_n = f(t_n)$$

Some other outputs currently available:

- $J = \left(P_g - P_{g,t}\right)^2$
- $J = \frac{1}{2}\left(A_e - A_{e,t}\right)\left(A_e - A_{e,t}\right)^T$

- Sonic boom discrete-adjoint equations:

sBOOMAdjoint:

$$\lambda_n^T = -\frac{\partial J_n}{\partial p_n} + \gamma_{0,n+1}^T k_{n+1} B^{n+1}$$

$$\beta_n^T A_3^n = \lambda_n^T \frac{\partial f_n}{\partial t_n}$$

$$\gamma_{1,n}^T A_2^n = \beta_n^T B_3^n$$

$$\gamma_{0,n}^T A^n = \gamma_{1,n}^T B_2^n$$

- Outputs/Objectives:

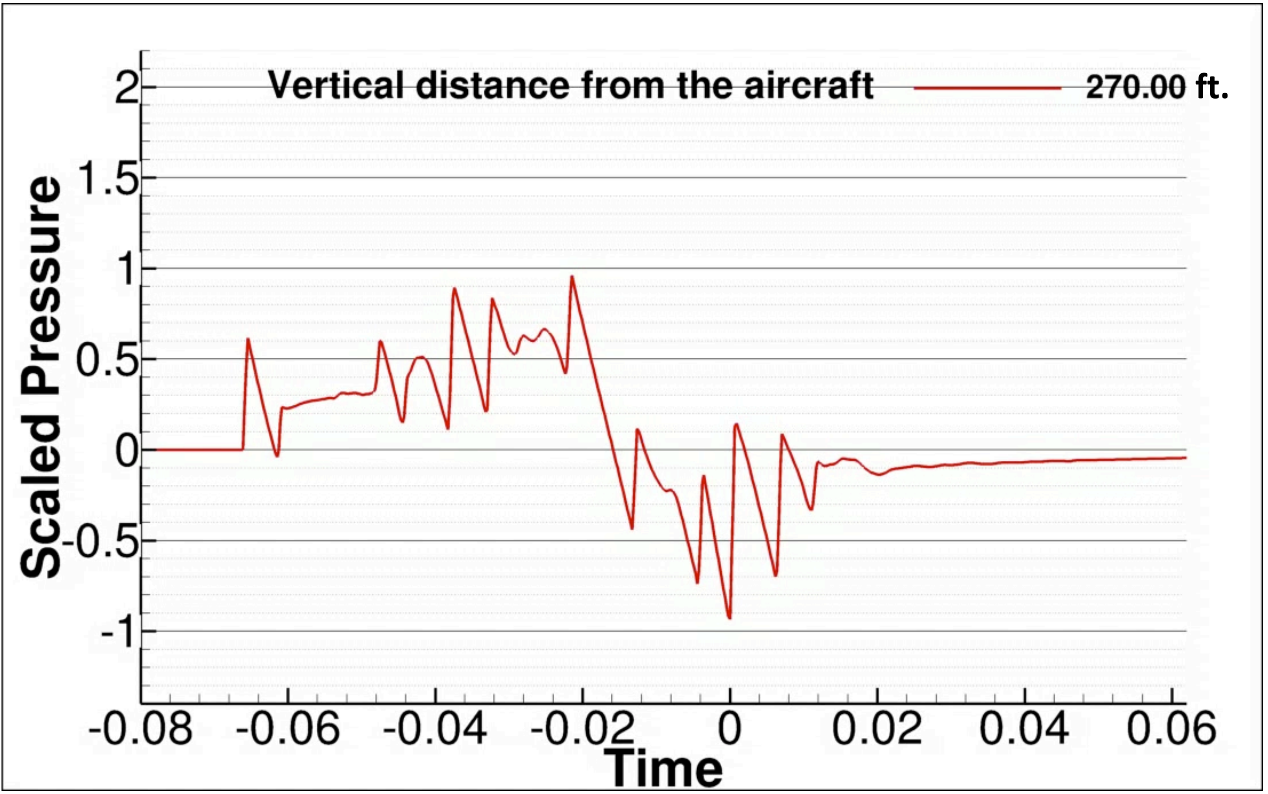
$$J = ASEL$$

$$\frac{\partial J}{\partial p_g} = \frac{\partial (ASEL)}{\partial p_g}$$

$$J = P_g^2$$

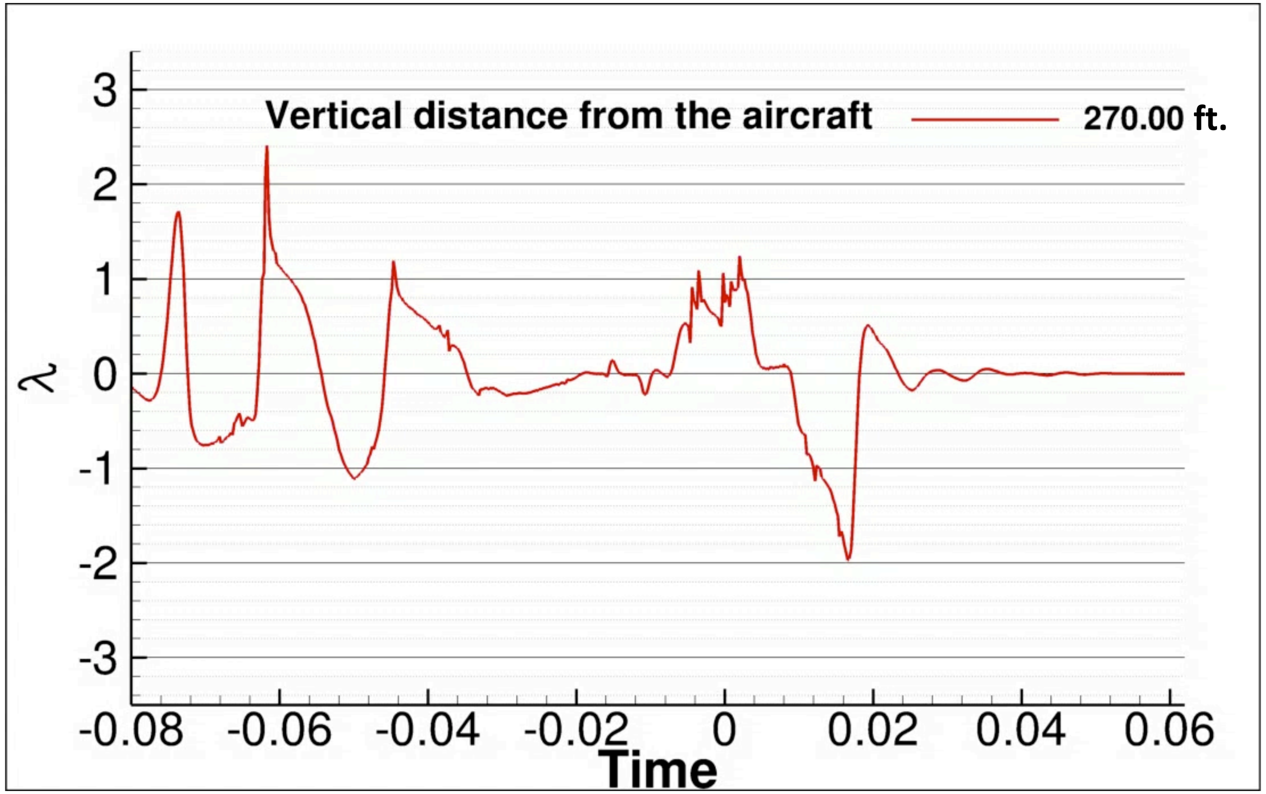
$$\frac{\partial J}{\partial p_g} = 2P_g$$

Sonic Boom Adjoints: Loudness

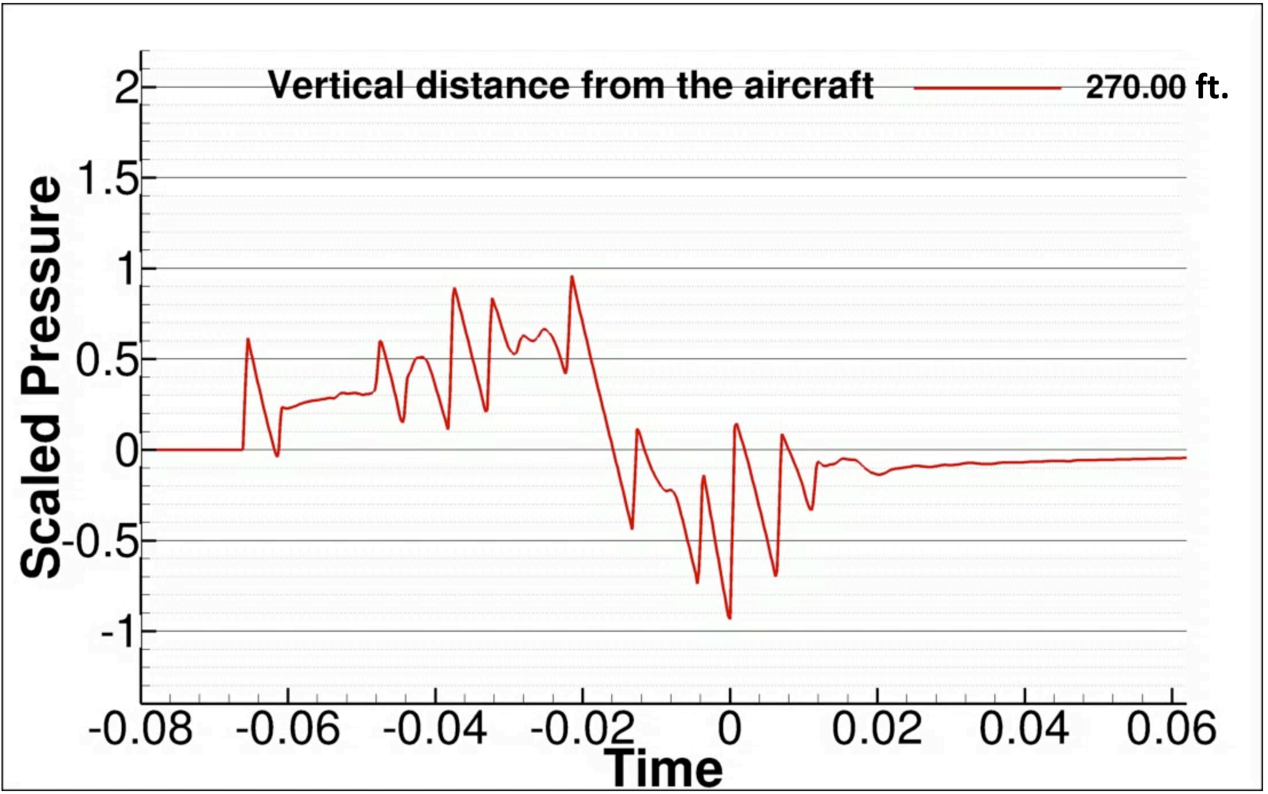


$$J = ASEL$$

$$\frac{\partial J}{\partial p_g} = \frac{\partial (ASEL)}{\partial p_g}$$

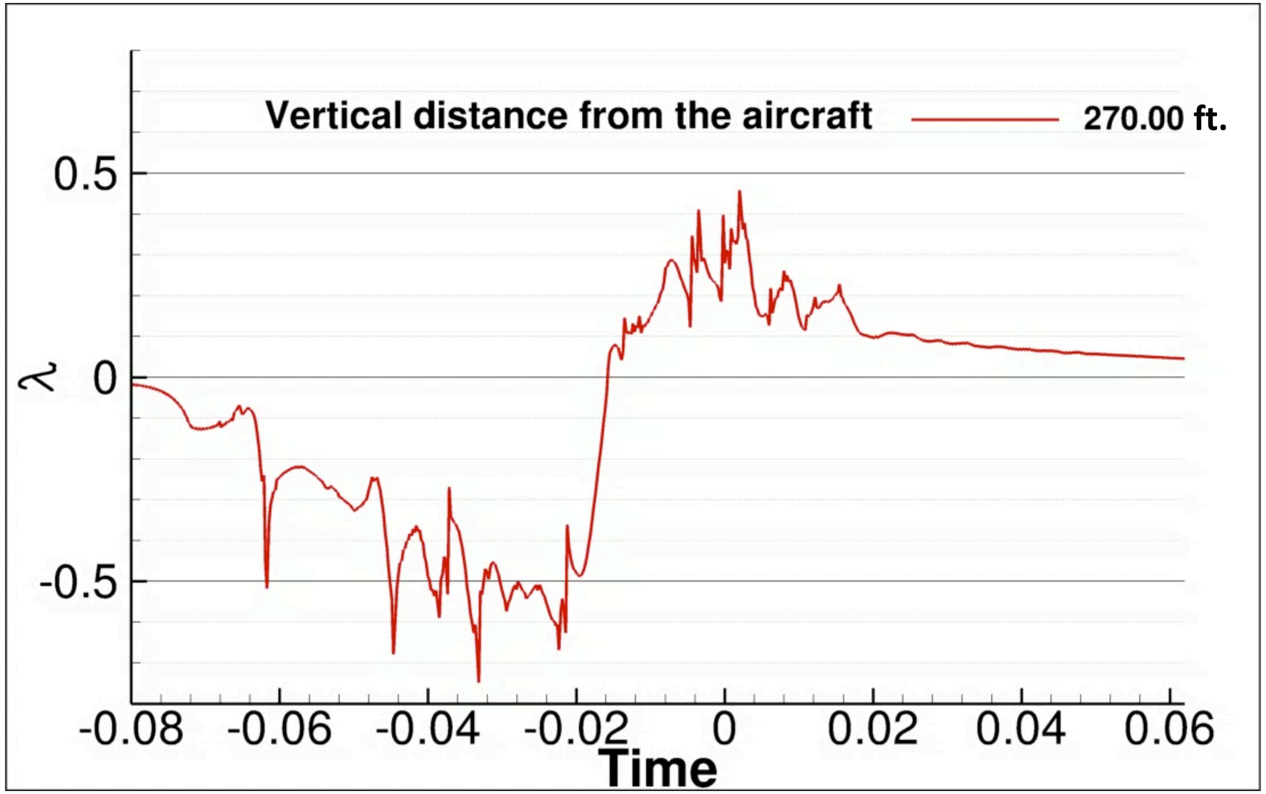


Sonic Boom Adjoints: Ground Pressures



$$J = P_g^2$$

$$\frac{\partial J}{\partial p_g} = 2P_g$$



Adjoint Sensitivities

- Adjoint sensitivities verified
  - Great agreement with those from complex variable approach

$J = ASEL$

Grid Point	Adjoint Gradient	Complex Gradient
2	-6.905038627775740	-6.90503861583487
100	-2.090800737003511	-2.09080073325145
1000	12.717298769483072	12.71729876557384
2000	-5.460241220665764	-5.460241220578294

$J = P_g^2$

Grid Point	Adjoint Gradient	Complex Gradient
2	-0.233976119396085	-0.233976119345164
100	1.759314455499879	1.759314455758305
1000	16.769371001777564	16.769371001735749
2000	-6.488476197116740	-6.4884761971747344

Adjoint Error Estimation

Residuals

$$\boldsymbol{R} = \begin{pmatrix} \vec{A}_n\vec{q}_n - k_n\vec{B}_n\vec{p}_{n-1} \\ \vec{A}_{n,2}\vec{r}_n - \vec{B}_{n,2}\vec{q}_n \\ \vec{A}_{n,3}\vec{t}_n - \vec{B}_{n,3}\vec{r}_n \\ \vec{p}_n - \vec{f}_n \end{pmatrix}$$

Solutions

$$\boldsymbol{u_n} = \begin{pmatrix} \vec{p}_n \\ \vec{q}_n \\ \vec{r}_n \\ \vec{t}_n \end{pmatrix}$$

Adjoints

$$\boldsymbol{\Gamma_n^T} = \left[\vec{\gamma}_{0,n}^T \;\; \vec{\gamma}_{1,n}^T \;\; \vec{\beta}_{0,n}^T \;\; \vec{\lambda}_{0,n}^T\right]$$

Truncation error estimate

$$\text{Adjoint Error Correction} \approx \sum_{n=1}^{N} (\Gamma_{h,n}^H)^T R(U_{h,n}^H)$$

Interpolation from coarse grid to fine grid

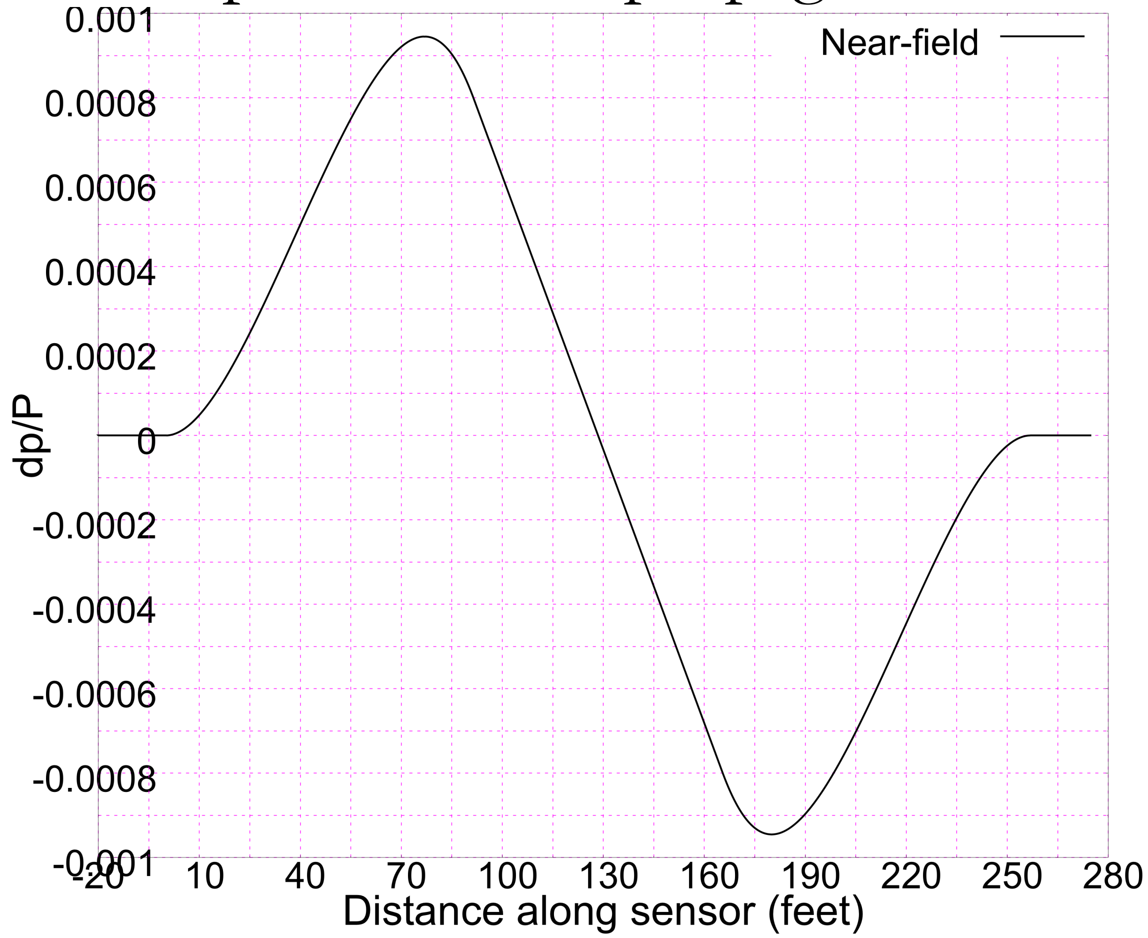
H = Coarse mesh

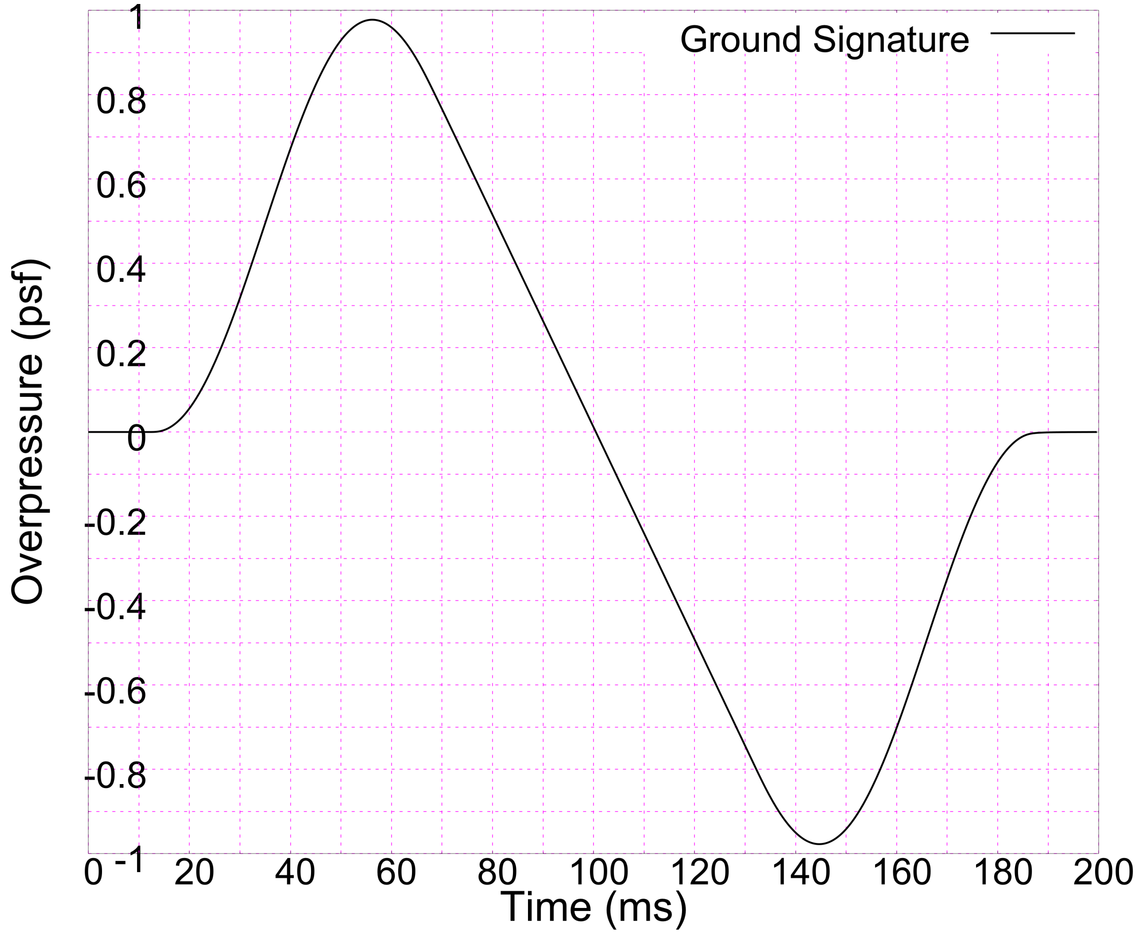
h = Embedded refined mesh

$$\text{Remaining Error} \approx \left|J_h - J_h^H + \sum_{n=1}^{N} (\Gamma_{h,n}^H)^T R(U_{h,n}^H)\right|$$

Results: Sine Wave

- Simple sine wave propagation, no shocks

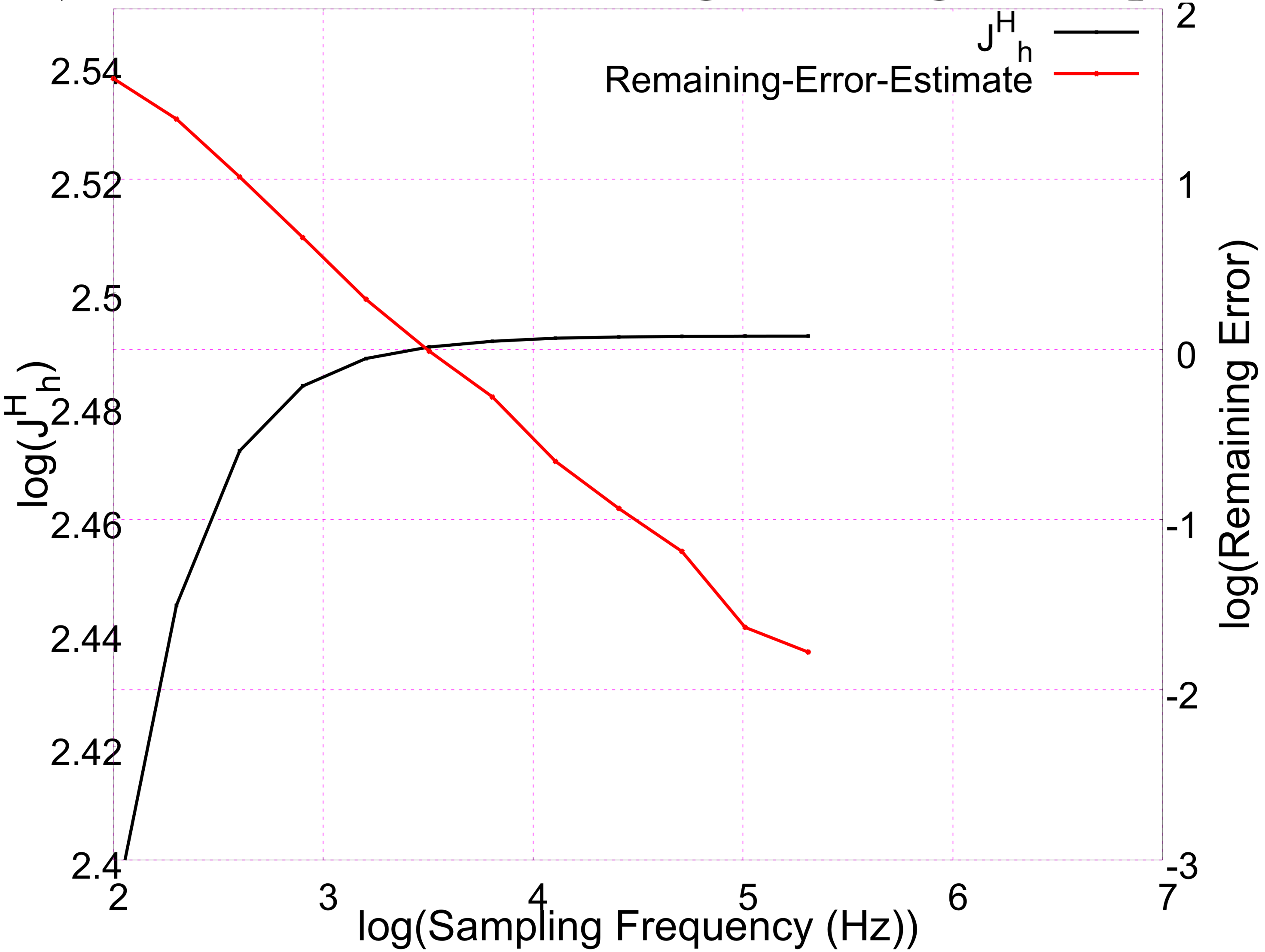




Results: Error in Ground Signatures, Sine Wave

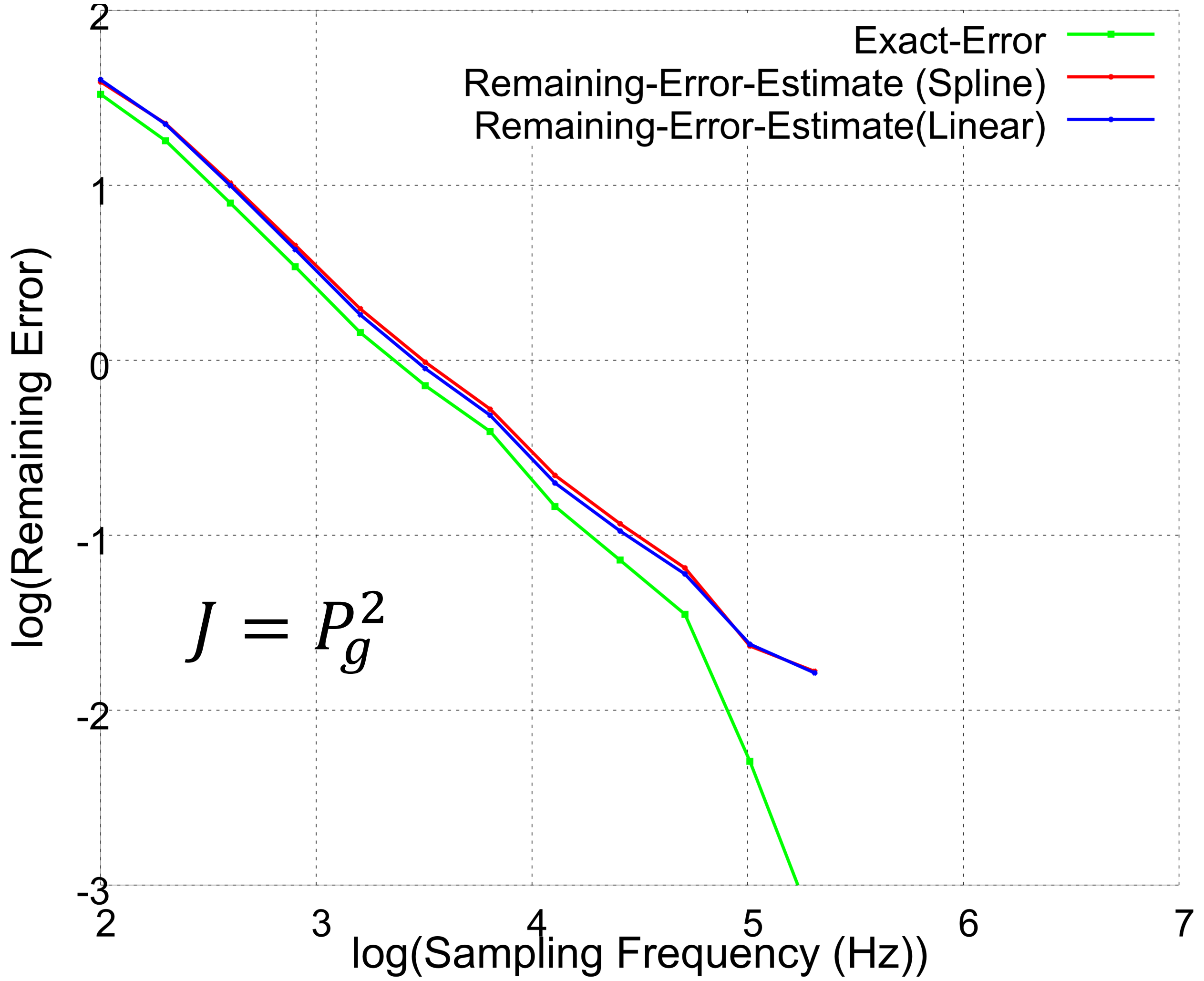
- Function and adjoint error estimation for ground signature pressures

$$J = P_g^2$$



Results: Error in Ground Signatures, Sine Wave

- Remaining error keeps dropping after adjoint error correction



Remaining Error Estimate

$$\left| J_h - J_h^H + \underbrace{\sum_{n=1}^{N} (\Gamma_{h,n}^H)^T R(U_{h,n}^H)}_{\text{Adjoint Correction}} \right|$$

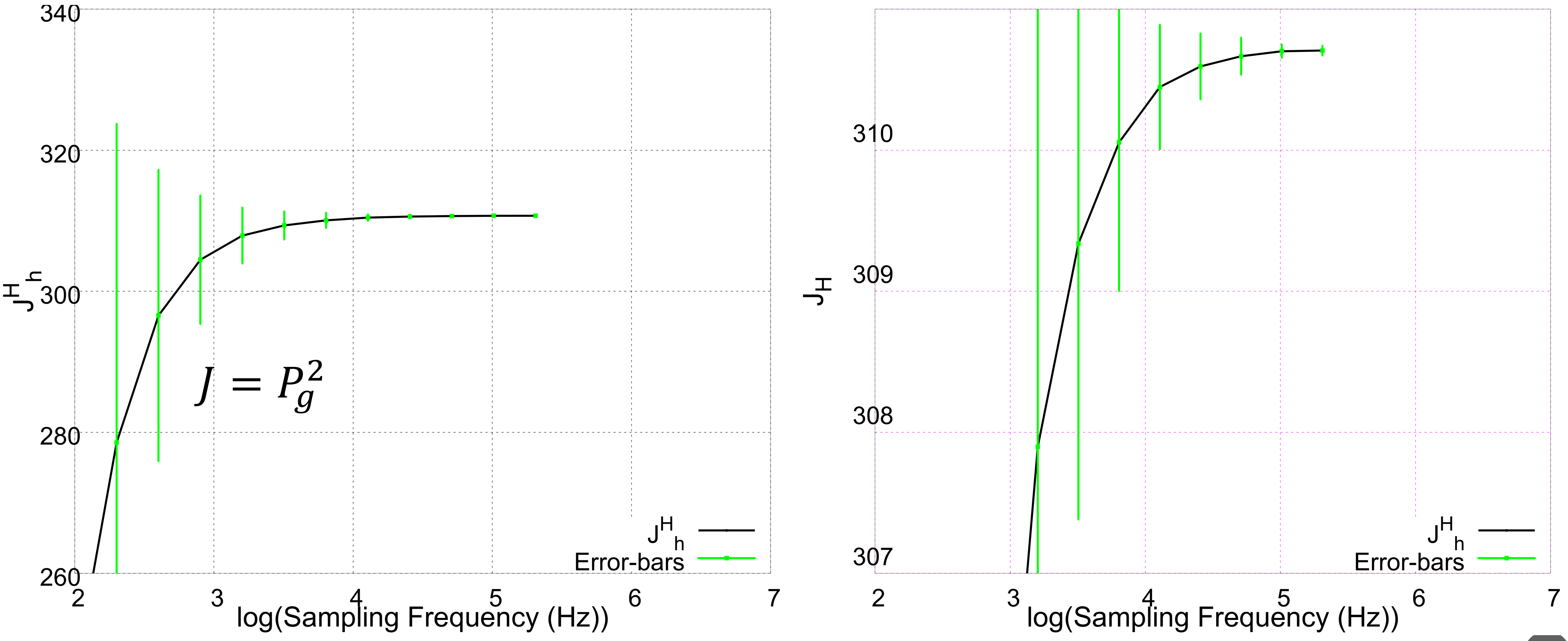
Adjoint Correction

Exact error

$$J_h - J_H$$

Results: Error in Ground Signatures, Sine Wave

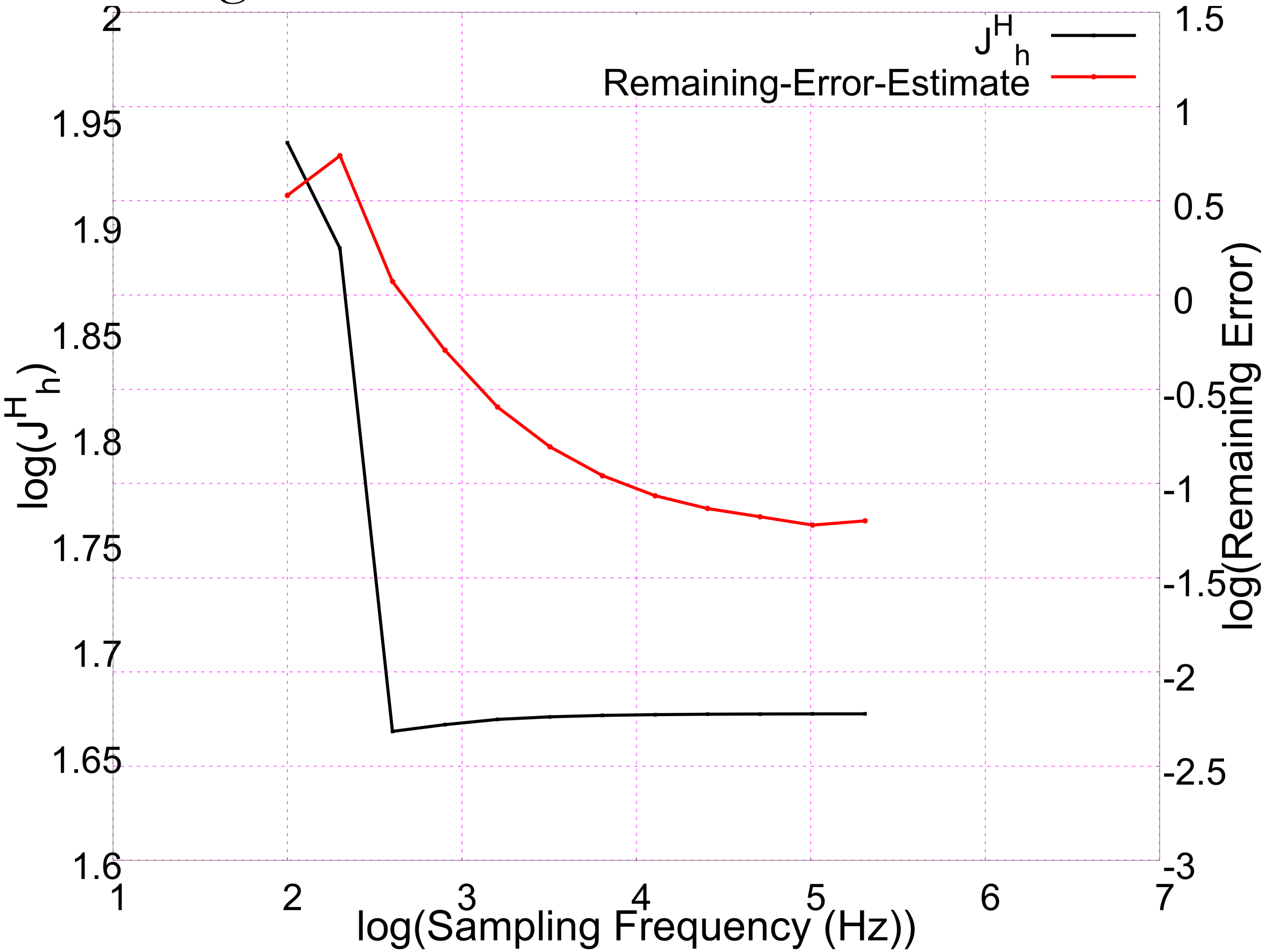
- Adjoint error bars keep decreasing with increasing sampling frequency



Results: Error in ASEL, Sine Wave

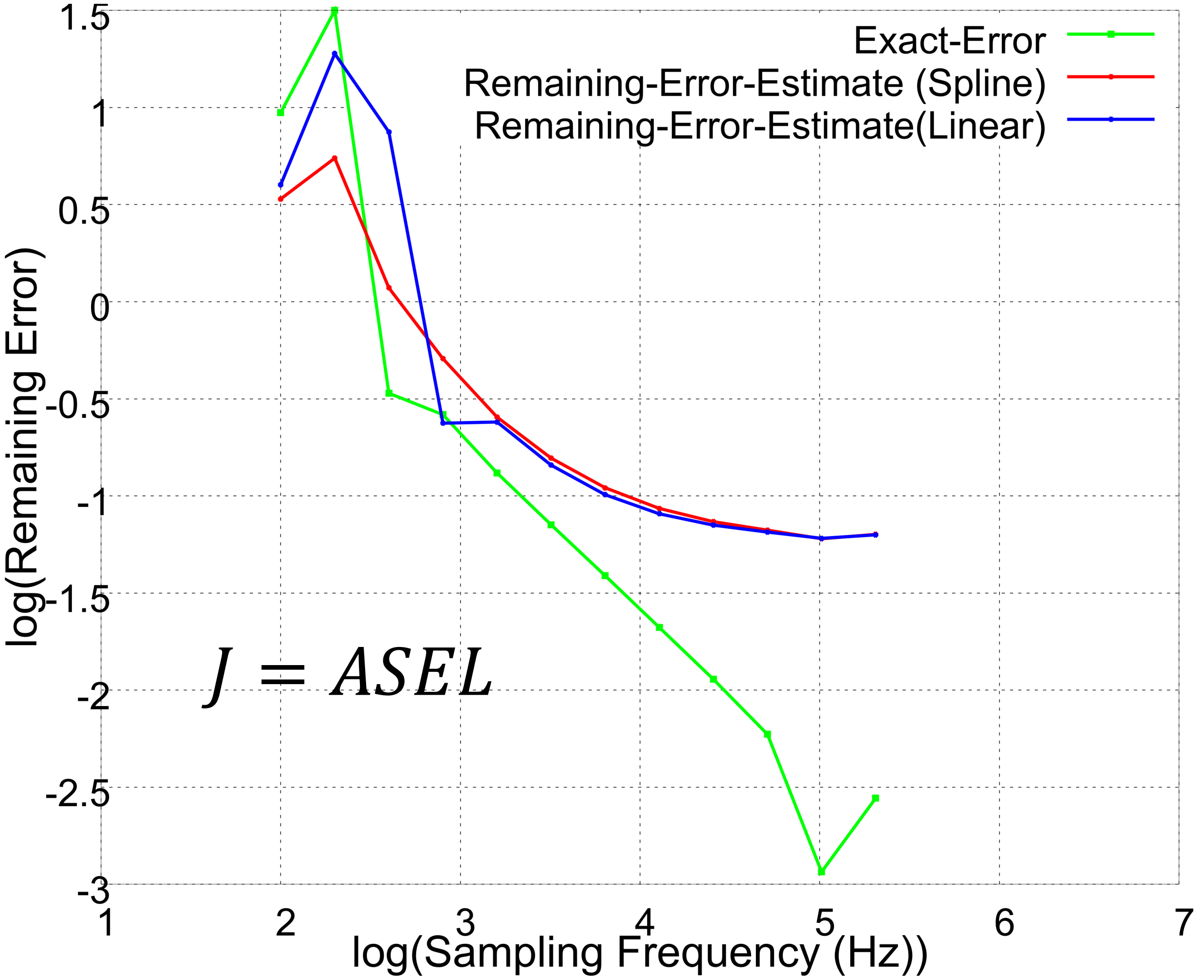
- Function and remaining error estimation for ASEL

$$J = ASEL$$



Results: Error in ASEL, Sine Wave

- Remaining error keeps dropping, but slowly



Remaining Error Estimate

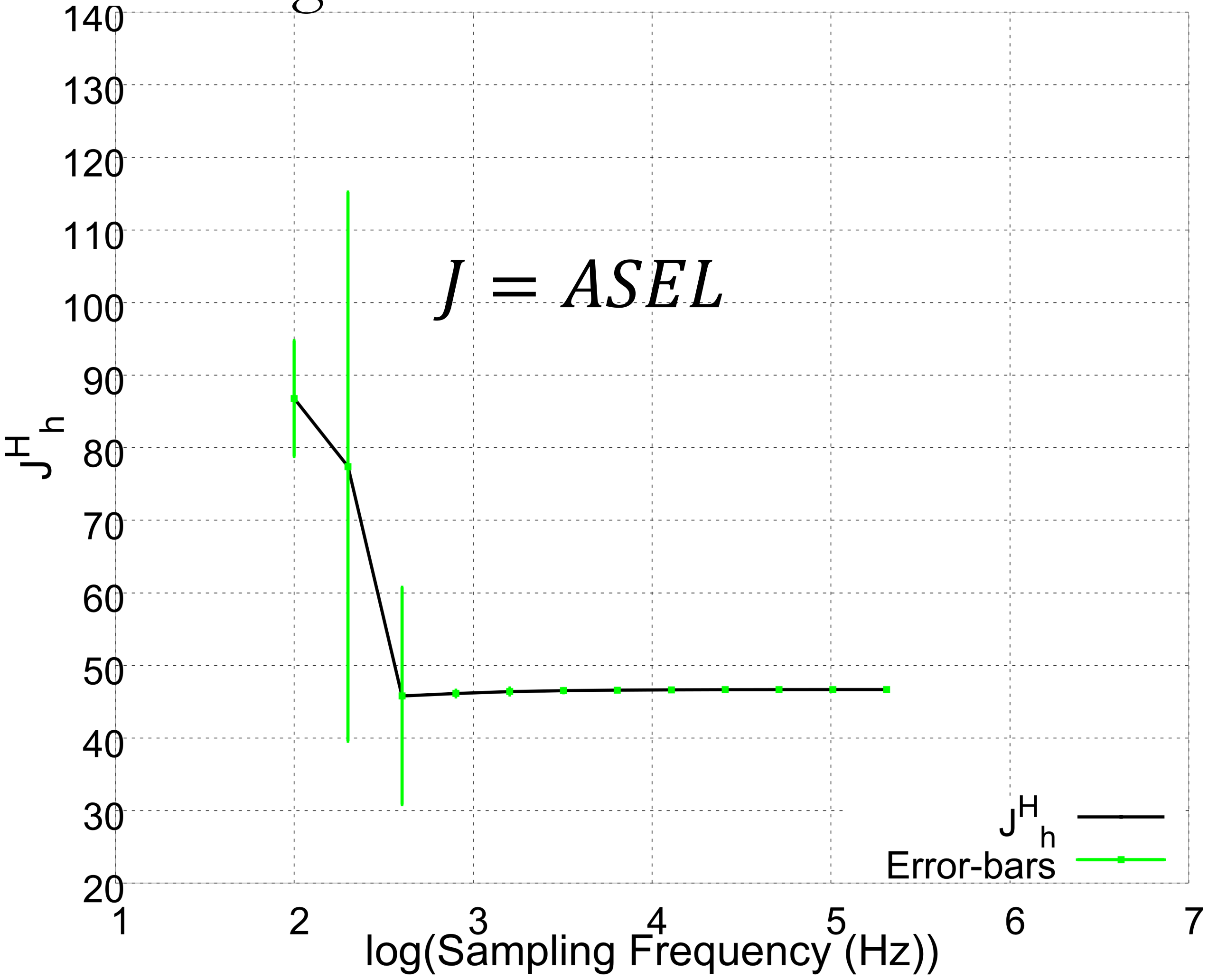
$$\left| J_h - J_h^H + \underbrace{\sum_{n=1}^{N} (\Gamma_{h,n}^H)^T R(U_{h,n}^H)}_{\text{Adjoint Correction}} \right|$$

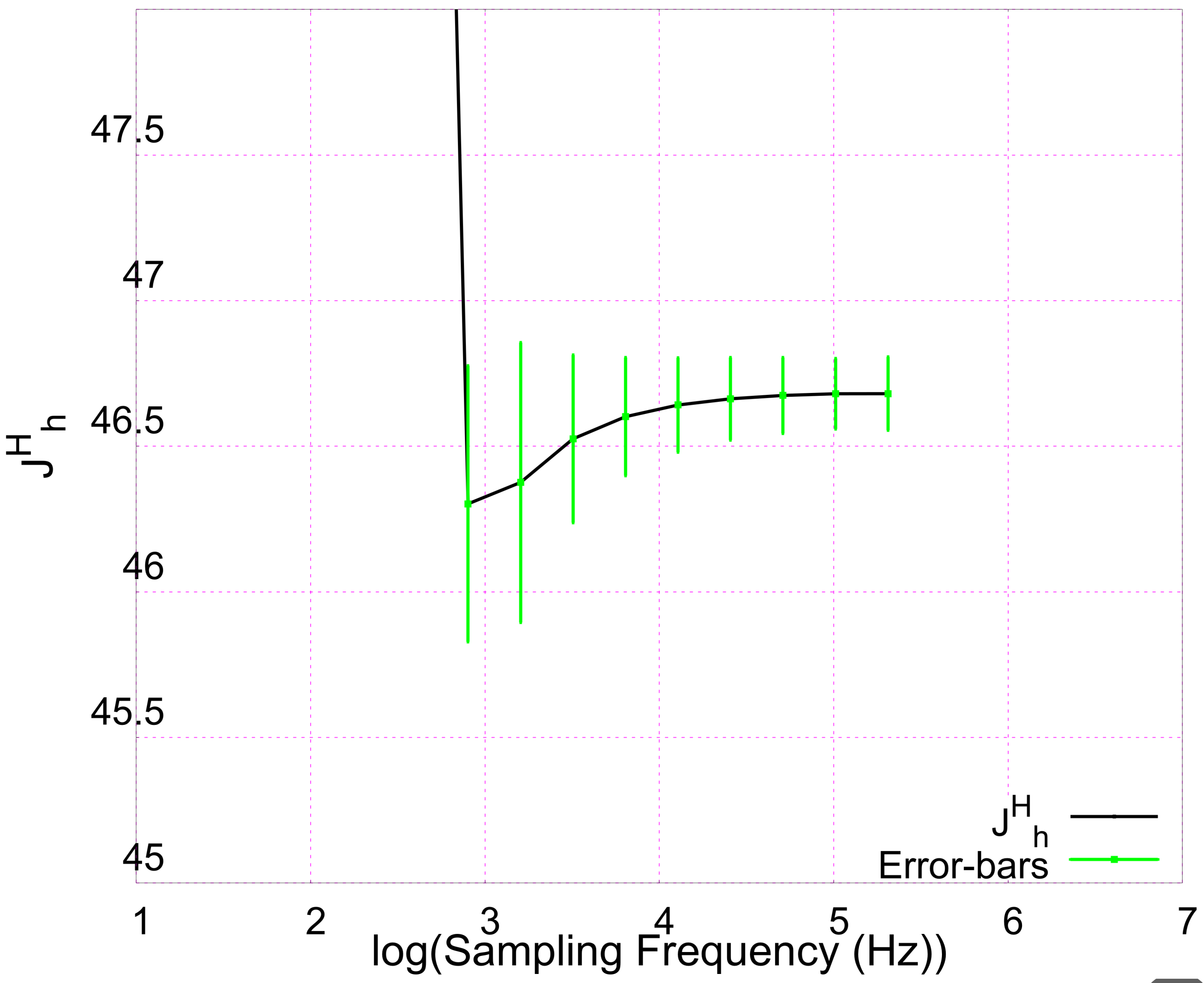
Exact error

$$J_h - J_H$$

Results: Error in ASEL, Sine Wave

- Remaining error decreasing at a much smaller rate, residuals not dropping fast enough





Summary and Future Work

- sBOOM enhanced to estimate error in sonic boom predictions by leveraging discrete adjoint methodology
- Errors useful in predicting how much variability exists in the current solution in determining metrics of interest
- Valuable to have *a priori* error estimates given the numerical discretization setup

Future Work

- Loudness calculation at high sampling frequencies
- Investigate error stalling and improve performance
- Reduce memory footprint and improve efficiency
- Verify estimated errors via Error Transport Equations (ETEs)

Acknowledgments

- NASA Commercial Supersonic Technology (CST) project
- Michael Park, NASA Langley
- Boris Diskin, NIA
- Michael Aftosmis/Marian Nemec, NASA Ames

Thank You! – Any Questions?

EXTRA SLIDES

Adjoint Error Estimation

H = Coarse mesh

h = Embedded refined mesh

Residual vector

$$\boldsymbol{R} = \begin{pmatrix} \vec{A}_n \vec{q}_n - k_n \vec{B}_n \vec{p}_{n-1} \\ \vec{A}_{n,2} \vec{r}_n - \vec{B}_{n,2} \vec{q}_n \\ \vec{A}_{n,3} \vec{t}_n - \vec{B}_{n,3} \vec{r}_n \\ \vec{p}_n - \vec{f}_n \end{pmatrix}$$

Solution vector

$$\boldsymbol{u_n} = \begin{pmatrix} \vec{p}_n \\ \vec{q}_n \\ \vec{r}_n \\ \vec{t}_n \end{pmatrix}$$

Adjoint vector

$$\boldsymbol{\Gamma_n^T} = \left[\vec{\gamma}_{0,n}^T \;\; \vec{\gamma}_{1,n}^T \;\; \vec{\beta}_{0,n}^T \;\; \vec{\lambda}_{0,n}^T \right]$$

$$J(U_h) \approx J\left(U_h^H\right) + \sum_{n=1}^{N} \frac{dJ\left(U_h^H\right)}{dU_{h,n}} \left(U_{h,n} - U_{h,n}^H\right)$$

$$R\left(U_{h,n}\right) = 0 \approx R\left(U_{h,n}^H\right) + \frac{dR\left(U_{h,n}^H\right)}{dU_{h,n}} \left(U_{h,n} - U_{h,n}^H\right)$$

$$J(U_h) \approx J\left(U_h^H\right) - \sum_{n=1}^{N} \frac{dJ\left(U_h^H\right)}{dU_{h,n}} \left[\frac{\partial R\left(U_{h,n}^H\right)}{\partial U_{h,n}}\right]^{-1} R\left(U_{h,n}^H\right)$$

$$\left(\Gamma_{h,n}^H\right)^T \frac{dR\left(U_{h,n}^H\right)}{dU_{h,n}} = \frac{dJ\left(U_h^H\right)}{dU_{h,n}}$$

$$J(U_h) \approx J\left(U_h^H\right) - \sum_{n=1}^{N} \left(\Gamma_{h,n}^H\right)^T R\left(U_{h,n}^H\right)$$

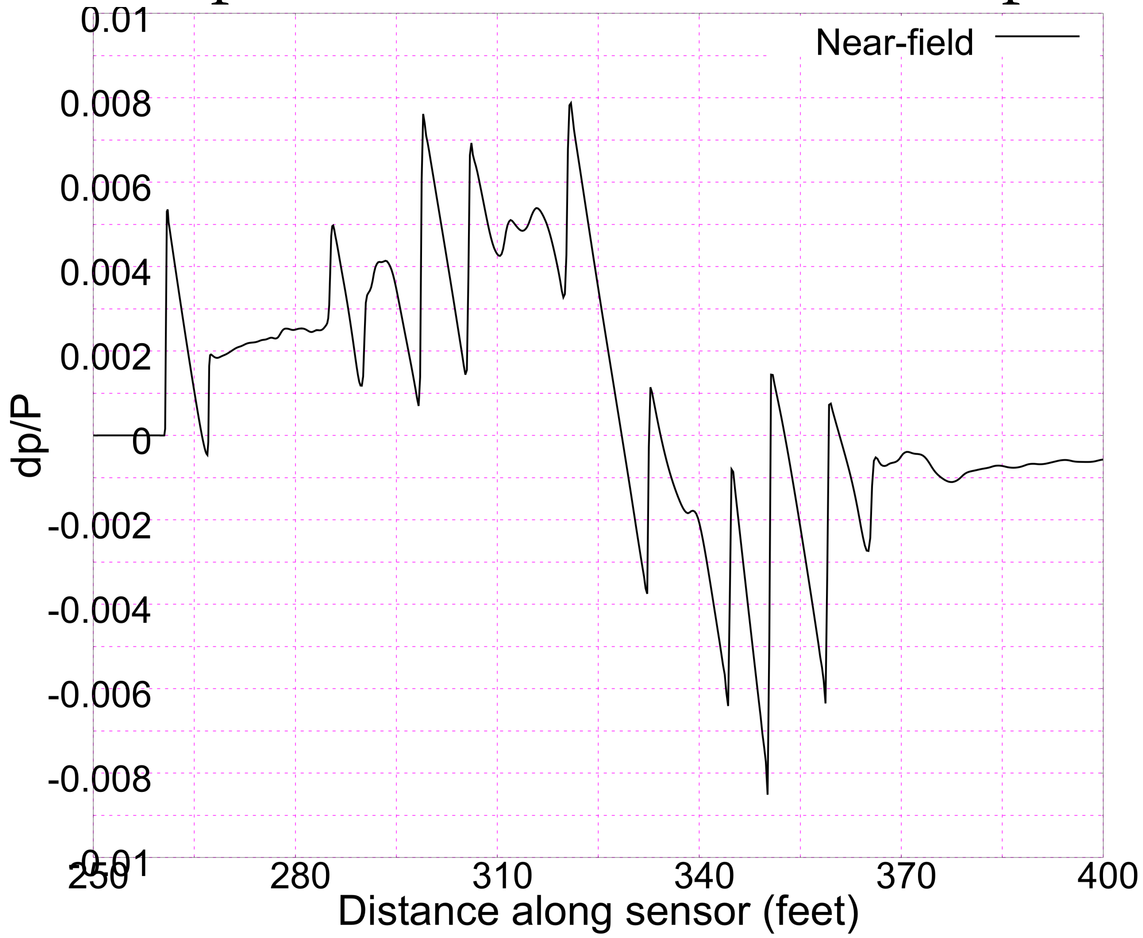
Adjoint Error Correction

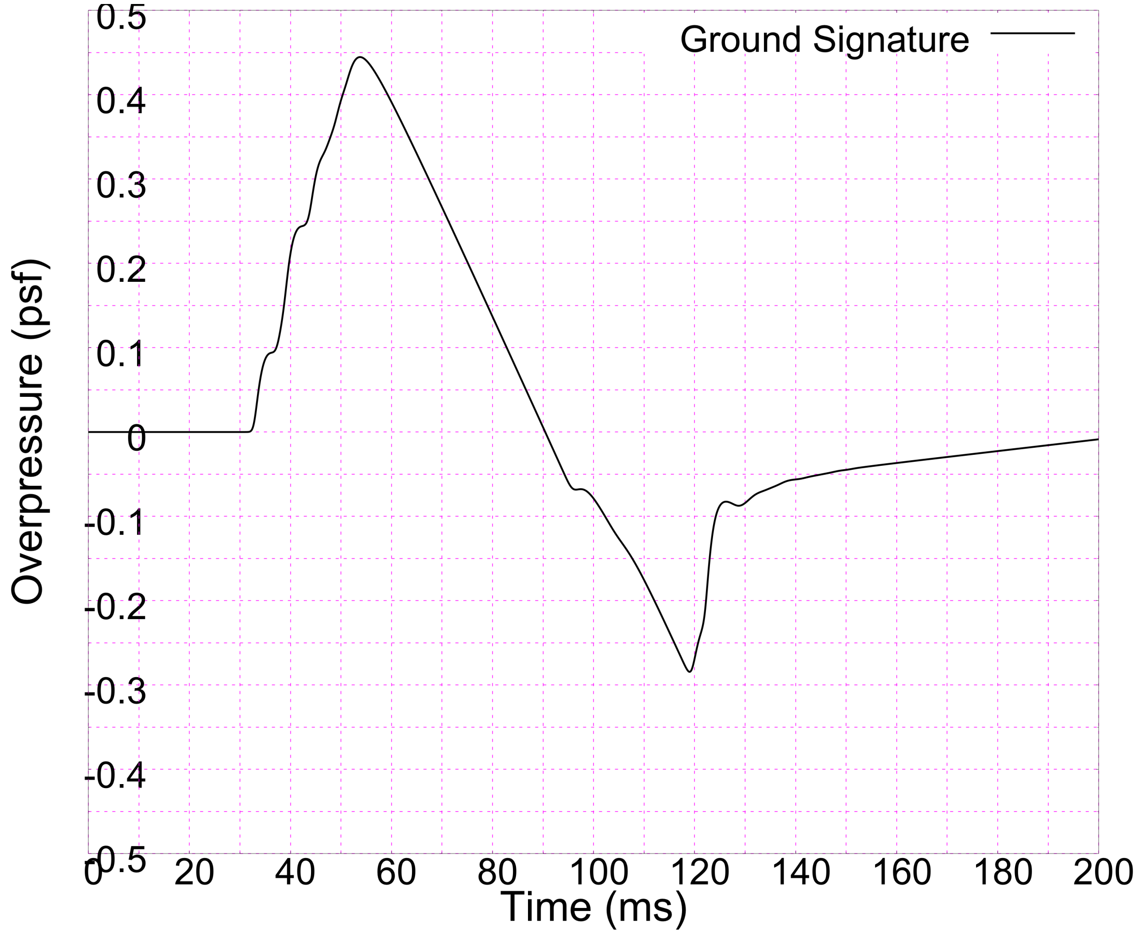
$$\approx \sum_{n=1}^{N} \left(\Gamma_{h,n}^H\right)^T R\left(U_{h,n}^H\right)$$

$$\text{Remaining Error} \approx \left| J_h - J_h^H + \sum_{n=1}^{N} \left(\Gamma_{h,n}^H\right)^T R\left(U_{h,n}^H\right) \right|$$

Results: Error in Loudness Metrics, Non-Sine Wave

- Shaped boom waveform, multiple shocks and dominant non-linear effects





Results: Error in Loudness Metrics, Non-Sine Wave

- Adjoint error estimate for loudness: ASEL