Application of Adjoint Methodology in Various Aspects of Sonic Boom Design

Sriram K. Rallabhandi, National Institute of Aerospace

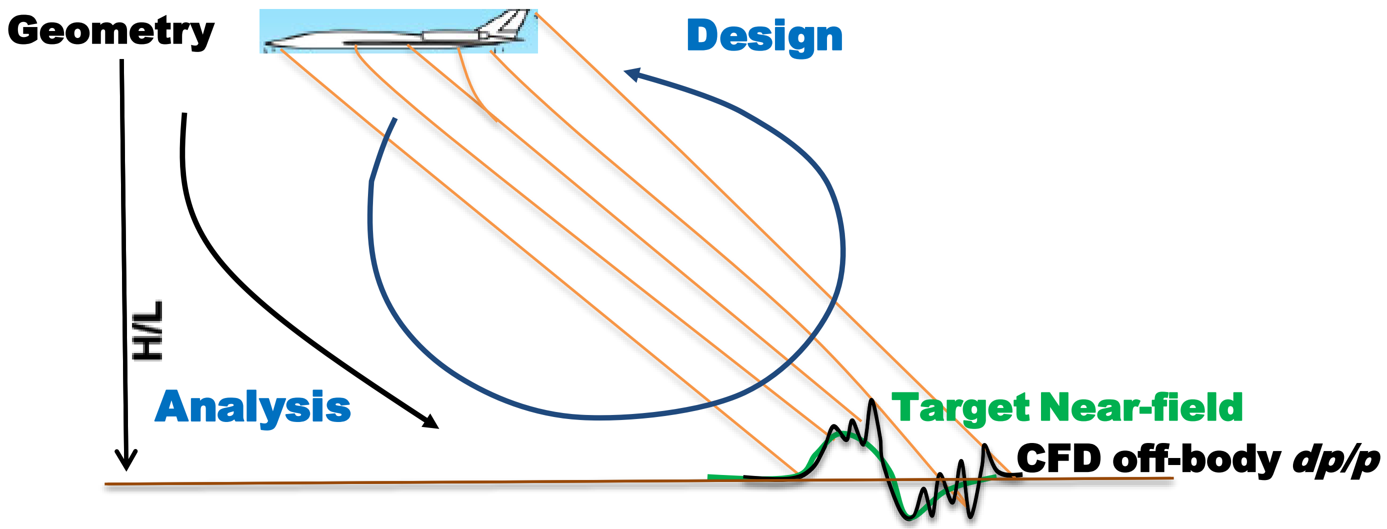
In support of NASA High Speed Project

- AIAA Aviation 2014
- Sonic Boom Activities III
- June 17, 2014

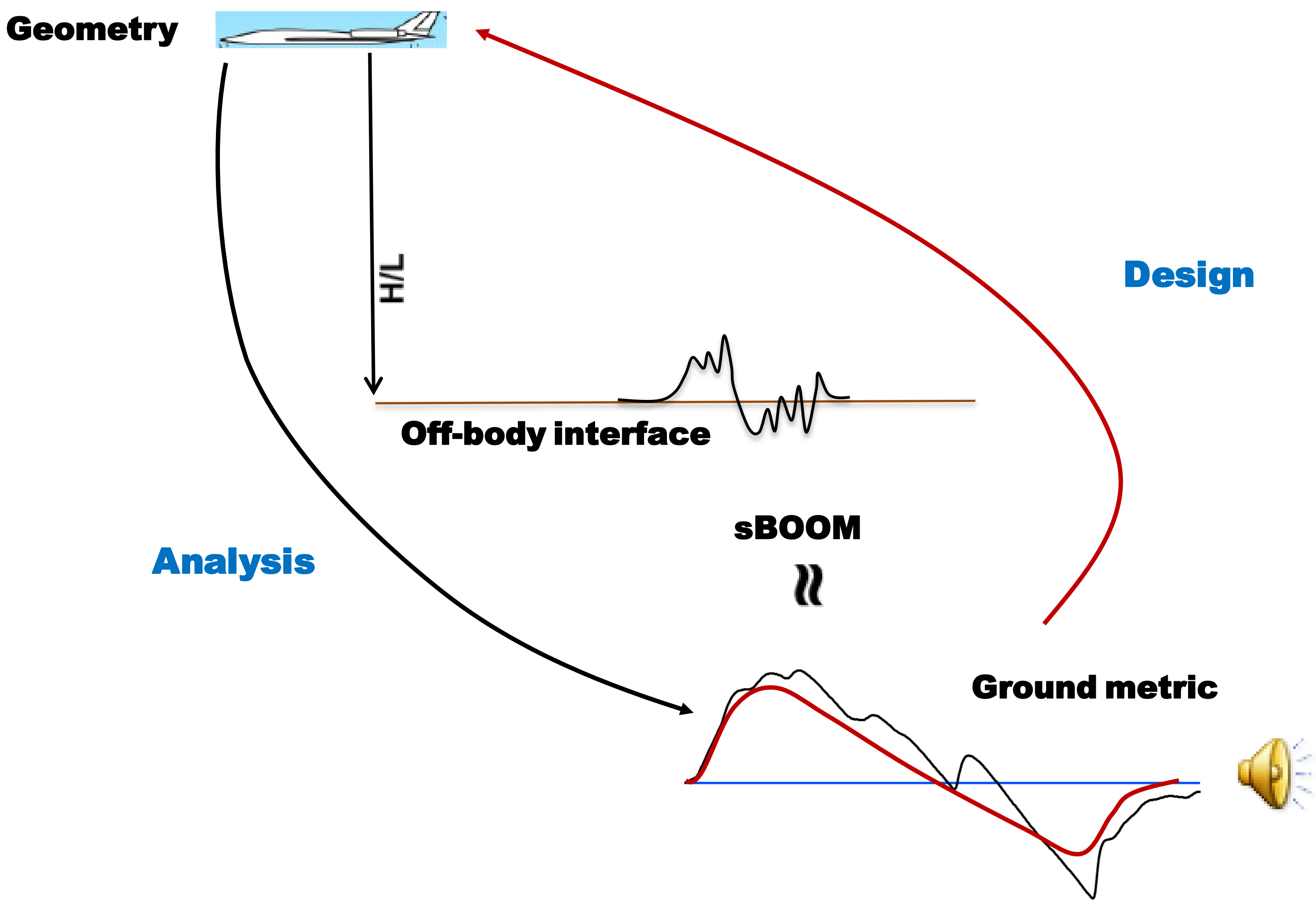
Outline

- Current status of adjoint shape optimization applied to sonic boom mitigation
- Goals
- Extension to boom adjoint theory
- Verification of adjoint sensitivities
- Results
- Discussion
- Summary

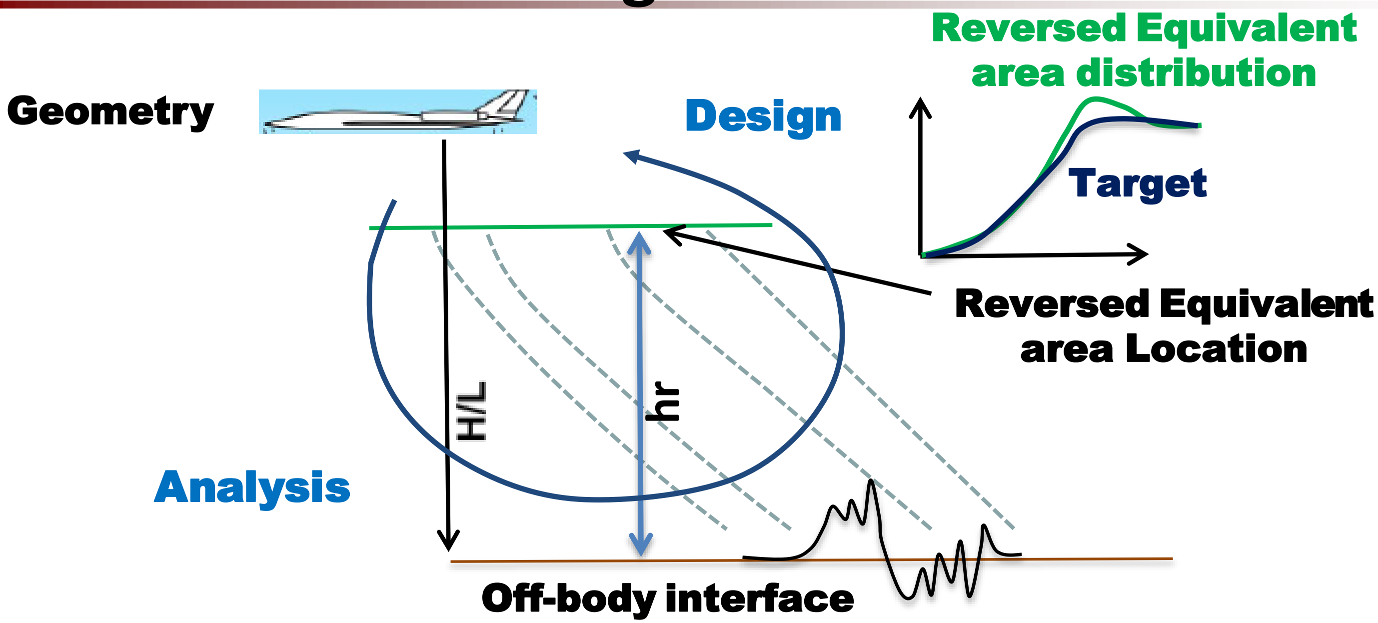
Current Status: Adjoint-Based Shape Optimization for Sonic Boom Mitigation



Current Status: Adjoint-Based Shape Optimization for Sonic Boom Mitigation



Current Status: Adjoint-Based Shape Optimization for Sonic Boom Mitigation



Goals

Extend sonic boom propagation utility (sBOOM[1,2]) to:

- Generate target equivalent areas using adjoint sensitivities
  - Targets at multiple azimuths in the neighborhood of the baseline design
  - Efficient compared to non-gradient based optimization approaches
- Obtain sensitivity of sonic boom metrics to:
  - Flight conditions
  - Propagation parameters
  - Atmospheric quantities

[1]Rallabhandi, S. K., "Advanced Sonic-Boom Prediction Using the Augmented Burgers Equation", Journal of Aircraft, Vol. 48, pp: 1245-1253, 2011

[2]Rallabhandi, S. K., Nielsen, E. J., Diskin, B., "Sonic-Boom Mitigation Through Aircraft Design and Adjoint Methodology", Journal of Aircraft, Vol. 51, pp: 502-510, 2014

Discrete Boom Adjoint Formulation

- Discrete-adjoint Lagrangian

$$D = P_0$$

$$L = \sum_{n=1}^{N} J_n(p,D) + \sum_{n=2}^{N}\gamma_{0,n}^T (A^n q_n - k_n B^n p_{n-1}) + \sum_{n=1}^{N}\gamma_{1,n}^T (A_2^n r_n - B_2^n q_n)$$
$$+ \sum_{n=1}^{N} \beta_n^T (A_3^n t_n - B_3^n r_n) + \sum_{n=1}^{N} \lambda_n^T (p_n - f(t_n)) + \gamma_{0,1}^T (A^1 q_1 - k_1 B^1 D)$$

sBOOM

$$A^n q_n = k_n B^n p_{n-1}$$
$$A_2^n r_n = B_2^n q_n$$
$$A_3^n t_n = B_3^n r_n$$
$$p_n = f(t_n)$$

- Derivative of the Lagrangian

$$\frac{dL}{dD} = \sum_{n=1}^{N}\left[\frac{\partial J_n}{\partial D} + \frac{\partial J_n}{\partial p_n}\frac{\partial p_n}{\partial D}\right] + \sum_{n=2}^{N}\gamma_{0,n}^T\left[A^n \frac{\partial q_n}{\partial D} - k_n B^n \frac{\partial p_{n-1}}{\partial D}\right]$$
$$+ \sum_{n=1}^{N}\gamma_{1,n}^T\left[A_2^n \frac{\partial r_n}{\partial D} - B_2^n \frac{\partial q_n}{\partial D}\right] + \sum_{n=1}^{N}\beta_n^T\left[A_3^n \frac{\partial t_n}{\partial D} - B_3^n \frac{\partial r_n}{\partial D}\right]$$
$$+ \sum_{n=1}^{N}\lambda_n^T\left[\frac{\partial p_n}{\partial D} - \frac{\partial f}{\partial t_n}\frac{\partial t_n}{\partial D}\right] + \gamma_{0,1}^T\left(A^1 \frac{\partial q_1}{\partial D} - k_1 B^1\right)$$

k_n – Blokhintzev scaling term
A^n, B^n – N_2 Relaxation matrices
A_2^n, B_2^n – O_2 Relaxation matrices
A_3^n, B_3^n – Absorption matrices
$f(t_n)$ – Nonlinear terms

Boom Adjoint Equations: Existing Theory

- Sonic boom discrete-adjoint equations:

$$\left.\begin{aligned} A^n q_n &= k_n B^n p_{n-1} \\ A_2^n r_n &= B_2^n q_n \\ A_3^n t_n &= B_3^n r_n \\ p_n &= f(t_n) \end{aligned}\right\} \text{sBOOM}$$

$$\left.\begin{aligned} \lambda_n^T &= -\frac{\partial J_n}{\partial p_n} + \gamma_{0,n+1}^T k_{n+1} B^{n+1} \\ \beta_n^T A_3^n &= \lambda_n^T \frac{\partial f_n}{\partial t_n} \\ \gamma_{1,n}^T A_2^n &= \beta_n^T B_3^n \\ \gamma_{0,n}^T A^n &= \gamma_{1,n}^T B_2^n \end{aligned}\right\} \text{sBOOMAdjoint}$$

- Cost functions:

$$J = (dBA - dBA_t)^2$$

$$\frac{\partial J}{\partial p_0} = 2(dBA - dBA_t)\frac{\partial (dBA)}{\partial p_0}$$

$$J = \tfrac{1}{2}(\overline{A_e} - \overline{A_{e,t}})(\overline{A_e} - \overline{A_{e,t}})^T$$

$$\frac{\partial J}{\partial p_o} = \left[\overline{A_e} - \overline{A_{e,t}}\right]\frac{\partial A_e}{\partial p_o}$$

- Gradient of the objective: $\frac{dL}{dD} = -\gamma_{0,1}^T k_1 B^1$

Boom Adjoints: Extension to Existing Theory

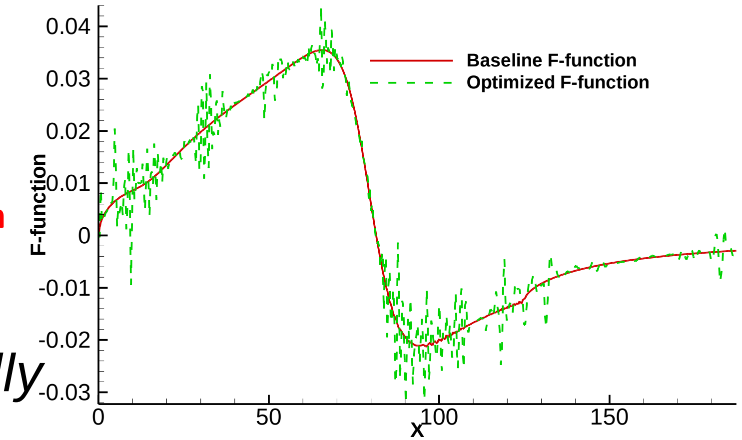
- Change of independent variable vector from p_0 to A_e

Existing formulation

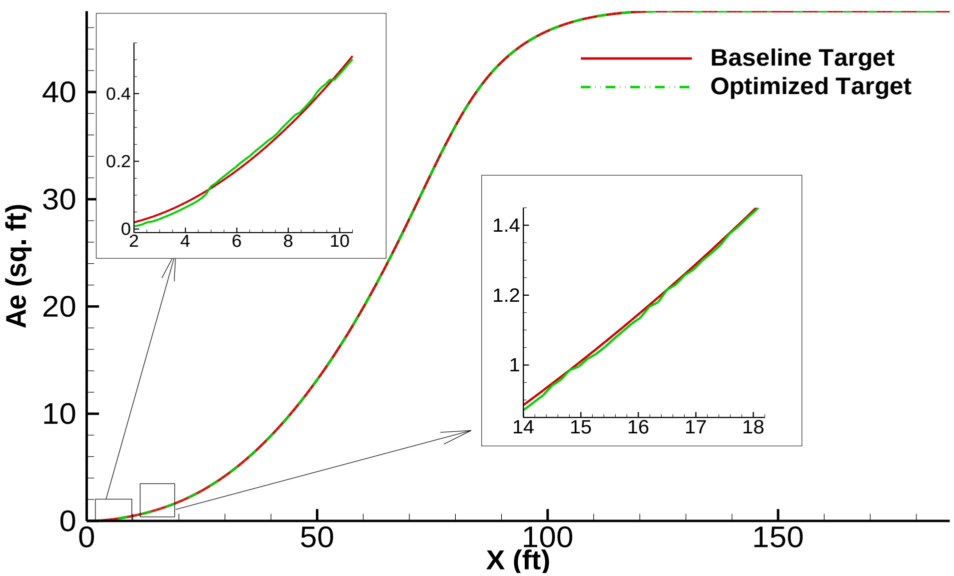
$$\underset{1xN}{\frac{dL}{dA_e}} = \underset{1xN}{\frac{dL}{d(p_0)}}\,\underset{NxN}{\frac{d(p_0)}{dA_e}} = \underset{1xN}{\frac{dL}{d(p_0)}}\,\underset{1x1}{\frac{\gamma M^2}{\sqrt{2\beta r}}}\,\underset{NxN}{\frac{dF}{dA_e}}$$

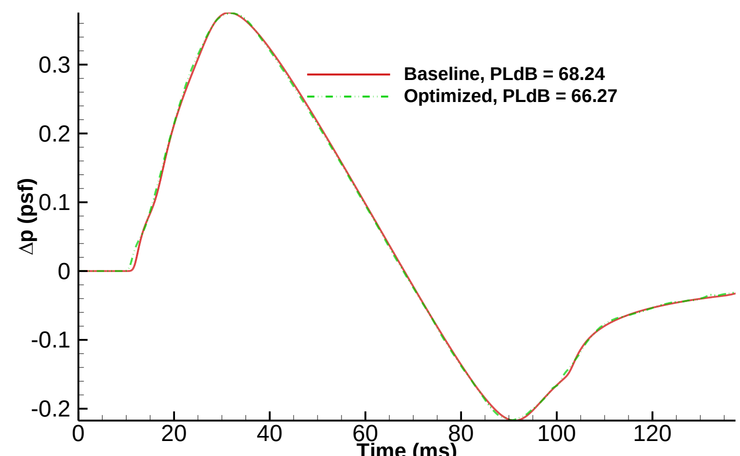
$$F(y) = \frac{1}{2\pi}\int_0^y \frac{A_e''}{(y-x)^{1/2}}\,dx$$

Numerical differentiation



- Gradient optimization leads to *numerically* better, but practically worse targets





Boom Adjoints: Extension to Existing Theory

- Change of independent variable vector from p_0 to A_e

Existing formulation

$$\underset{\text{1xN}}{\frac{dL}{dA_e}} = \underset{\text{1xN}}{\frac{dL}{d(p_0)}}\,\underset{\text{NxN}}{\frac{d(p_0)}{dA_e}} = \underset{\text{1xN}}{\frac{dL}{d(p_0)}}\,\underset{\text{1x1}}{\frac{\gamma M^2}{\sqrt{2\beta r}}}\,\underset{\text{NxN}}{\frac{dF}{dA_e}}$$

$$F(y) = \frac{1}{2\pi}\int_0^y \frac{A_e''}{(y-x)^{1/2}}\,dx$$

Numerical differentiation

- Smoothing is needed
  - ➢ Cubic spline based A_e targets based on control points
  - ➢ Algorithm not only returns spline interpolation, but also the Jacobian matrices: $\underset{\text{NxC}}{\frac{\partial A_e}{\partial X_{CP}}}, \underset{\text{NxC}}{\frac{\partial A_e}{\partial A_{e,CP}}}$ [$N > C$, CP = Control Points]
  - ➢ Gradient computation extended by chain rule

$$\underset{\text{1xC}}{\frac{dL}{dD}} = \begin{cases} \frac{dL}{dA_e}\frac{dA_e}{dX_{CP}} & \text{if } D = X_{CP} \\ \underset{\text{1xN}}{\frac{dL}{dA_e}}\,\underset{\text{NxC}}{\frac{dA_e}{dA_{e,CP}}} & \text{if } D = A_{e,CP} \end{cases}$$

Sensitivities wrt flight conditions and propagation parameters

- Independent variable vector D:$[\Delta\sigma, S, \Gamma, \theta_{v,1}, C_{v,1}, \theta_{v,2}, C_{v,2}, h, M]$
- Updated derivative of the Lagrangian

$$\underset{\text{1xM}}{\frac{dL}{dD}} = \sum_{n=1}^{N}\left[\underset{\text{1xM}}{\frac{\partial J_n}{\partial D}} + \underset{\text{1xN}}{\frac{\partial J_n}{\partial p_n}}\underset{\text{NxM}}{\frac{\partial p_n}{\partial D}}\right] + \sum_{n=1}^{N}\underset{\text{1xN}}{\gamma_{0,n}^{T}}\left[\underset{\text{NxN}}{A^n}\underset{\text{NxM}}{\frac{\partial q_n}{\partial D}} + \underset{\text{1xN}}{q_n^T}\underset{\text{NxNxM}}{\frac{\partial A^n}{\partial D}} - \underset{\text{1x1}}{k_n}\underset{\text{NxN}}{B^n}\underset{\text{NxM}}{\frac{\partial p_{n-1}}{\partial D}} - \underset{\text{1x1}}{k_n}\underset{\text{1xN}}{p_{n-1}^T}\underset{\text{NxNxM}}{\frac{\partial B^n}{\partial D}} - \underset{\text{NxN}}{B^n}\underset{\text{Nx1}}{p_{n-1}}\underset{\text{1xM}}{\frac{\partial k_n}{\partial D}}\right]$$

$$+\sum_{n=1}^{N}\underset{\text{1xN}}{\gamma_{1,n}^{T}}\left[\underset{\text{NxN}}{A_2^n}\underset{\text{NxM}}{\frac{\partial r_n}{\partial D}} + \underset{\text{1xN}}{r_n^T}\underset{\text{NxNxM}}{\frac{\partial A_2^n}{\partial D}} - \underset{\text{NxN}}{B_2^n}\underset{\text{NxM}}{\frac{\partial q_n}{\partial D}} - \underset{\text{1xN}}{q_n^T}\underset{\text{NxNxM}}{\frac{\partial B_2^n}{\partial D}}\right] + \sum_{n=1}^{N}\underset{\text{1xN}}{\beta_n^T}\left[\underset{\text{NxN}}{A_3^n}\underset{\text{NxM}}{\frac{\partial t_n}{\partial D}} + \underset{\text{1xN}}{t_n^T}\underset{\text{NxNxM}}{\frac{\partial A_3^n}{\partial D}} - \underset{\text{NxN}}{B_3^n}\underset{\text{NxM}}{\frac{\partial r_n}{\partial D}} - \underset{\text{1xN}}{r_n^T}\underset{\text{NxNxM}}{\frac{\partial B_3^n}{\partial D}}\right]$$

$$+\sum_{n=1}^{N}\underset{\text{1xN}}{\lambda_n^T}\left[\underset{\text{NxM}}{\frac{\partial p_n}{\partial D}} - \underset{\text{NxN}}{\frac{\partial f}{\partial t_n}}\underset{\text{NxM}}{\frac{\partial t_n}{\partial D}}\right]$$

Previous gradient computation: $\frac{dL}{dD} = \gamma_{0,1}^{T} k_1 B^1$

- Updated gradient calculation:

$$\frac{dL}{dD} = \sum_{n=1}^{N}\gamma_{0,n}^{T}\left[q_n^T\frac{\partial A^n}{\partial D} - k_n p_{n-1}^T\frac{\partial B^n}{\partial D} - B^n p_{n-1}\frac{\partial k_n}{\partial D}\right]$$

$$+\sum_{n=1}^{N}\gamma_{1,n}^{T}\left[r_n^T\frac{\partial A_2^n}{\partial D} - q_n^T\frac{\partial B_2^n}{\partial D}\right] + \sum_{n=1}^{N}\beta_n^T\left[t_n^T\frac{\partial A_3^n}{\partial D} - r_n^T\frac{\partial B_3^n}{\partial D}\right]$$

- Significant increase in memory requirement (~12 GB more than previous formulation for a typical case)

Verification of Adjoint Sensitivities

- Adjoint sensitivities verified against complex step gradients
  - Imaginary step size of 10^{-50} used in evaluating complex gradients
  - Good match up to 8 digits of numerical accuracy for target A_e

Grid Point	Adjoint Gradient	Complex Gradient
0	4.99555122854739	4.99555122485994
200	0.22333517023795	0.223335170026696
500	-1.83336776546880	-1.83336776671720

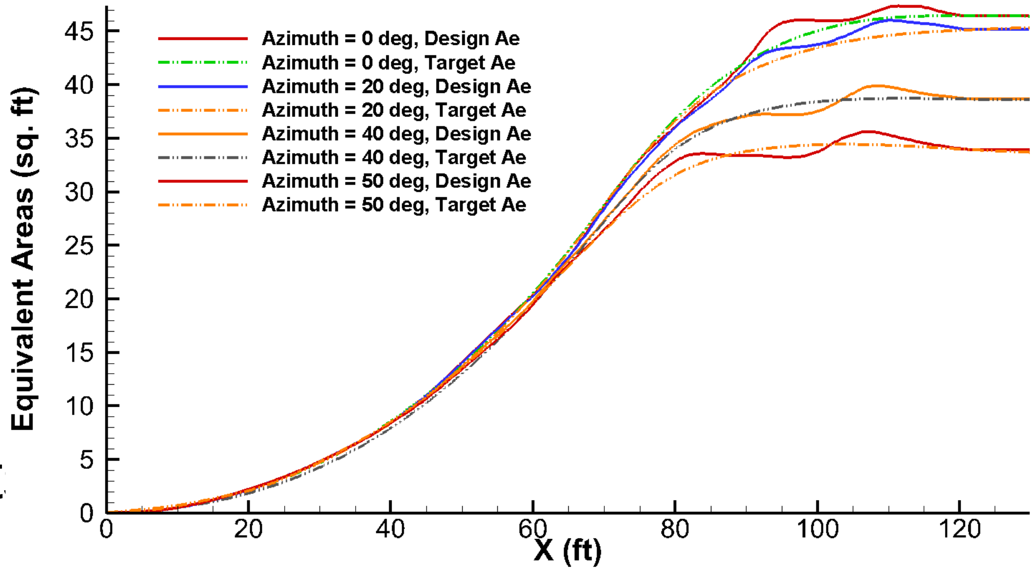
- Sensitivities wrt flight conditions, and propagation parameters

Variable	Adjoint Gradient	Complex Gradient
S	-0.121609572658E-002	-0.121609679831E-002
$\Delta\sigma$	-0.101433622238E+004	-0.101434351727E+004
Γ	-0.242603369041E+005	-0.242607576473E+005
C_{ν_1}	0.850297505015E+002	0.850293279821E+002
C_{ν_2}	0.114275582996E+003	0.114279749352E+003
θ_{ν_1}	-0.270434382922E+004	-0.270438763849E+004
θ_{ν_2}	-0.226512310051E+005	-0.226534988755E+005
M	0.103286198407E+002	0.103281674303E+002
h	0.373155218390E-004	0.373155285384E-004

Sriram.Rallabhandi@nasa.gov

Results: Target A_e Generation

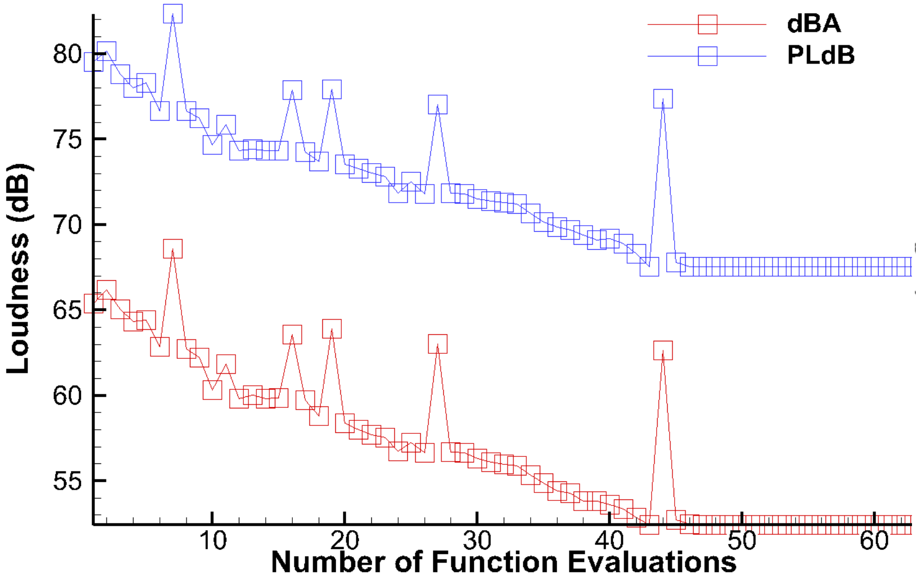
- 15 spline control points
- SQP optimization with A-weighted loudness (dBA) to be minimized
- 64-bit Xeon CPU with 16GB memory
- Total wall time ~ 30 minutes
- Under- and off-track targets generated simultaneously
- Targets generated in the neighborhood of the baseline

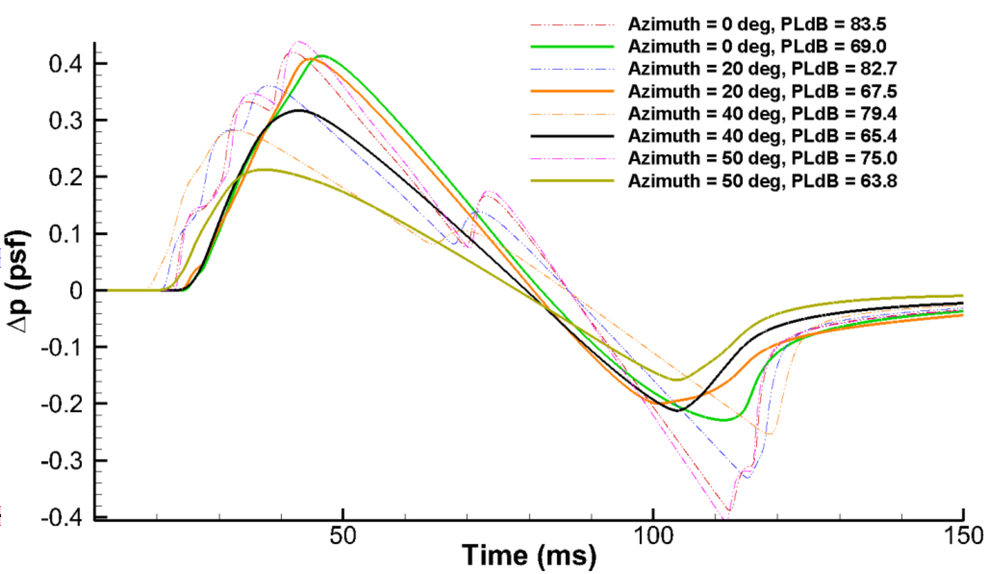


Sriram.Rallabhandi@nasa.gov

Results: Target A_e generation

- Typically convergence achieved in 50-60 function evaluations
- A-weighted loudness (dBA) minimized, but perceived loudness tracked
  - Good correlation between dBA and PLdB

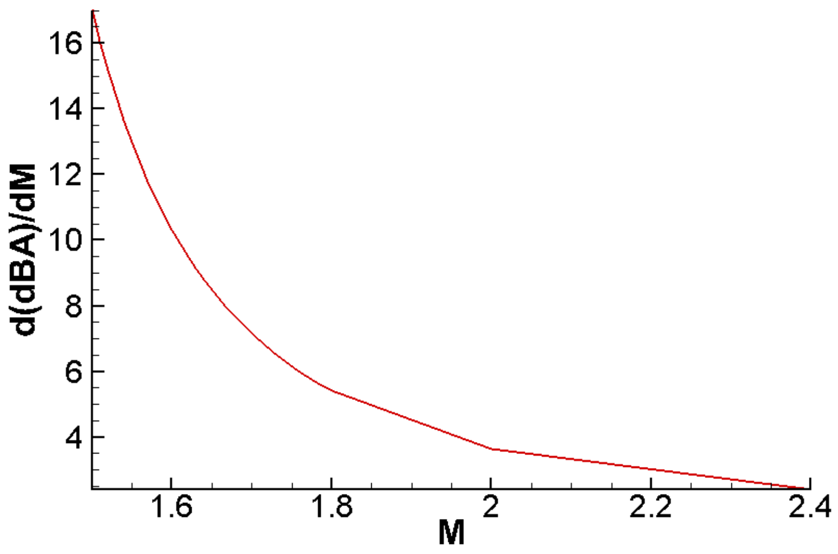


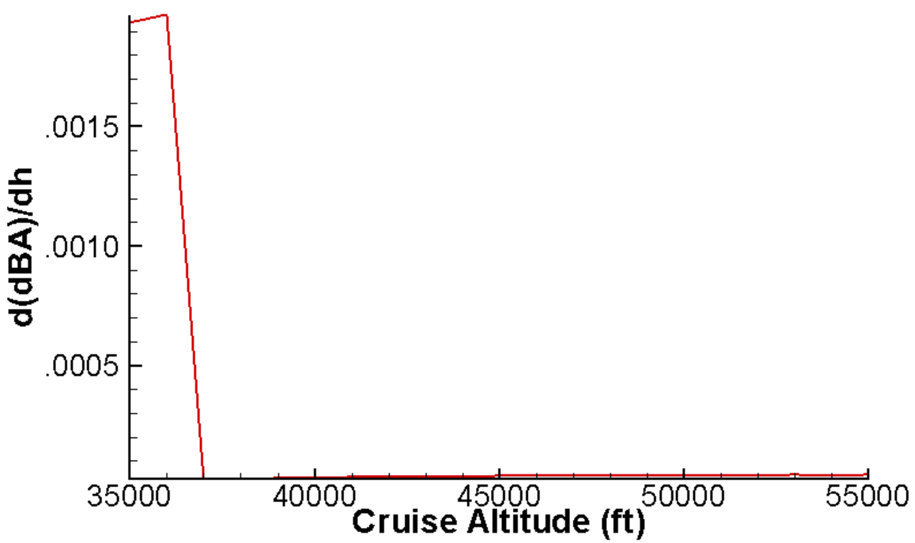


Sriram.Rallabhandi@nasa.gov

Results: Atmospheric Sensitivity

- Loudness sensitivity to Mach number decreases with Mach number
- Loudness sensitivity to cruise altitude approaches zero in tropopause

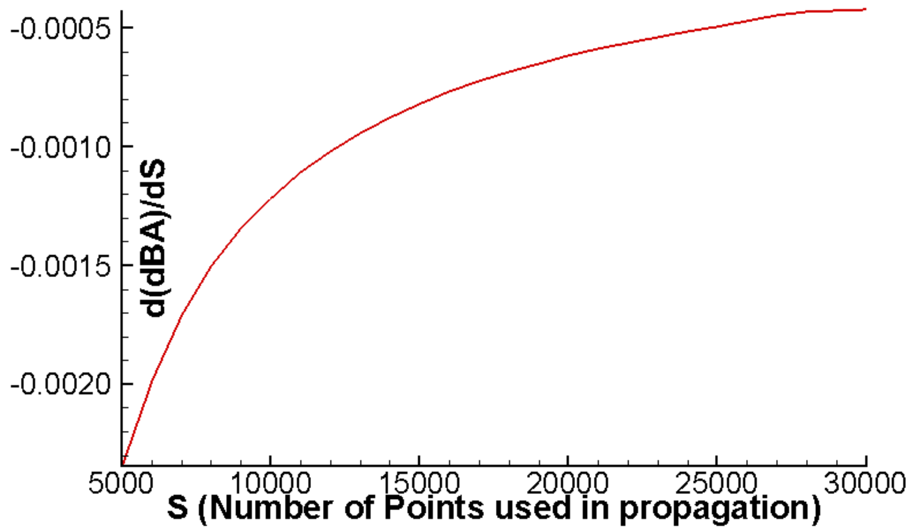


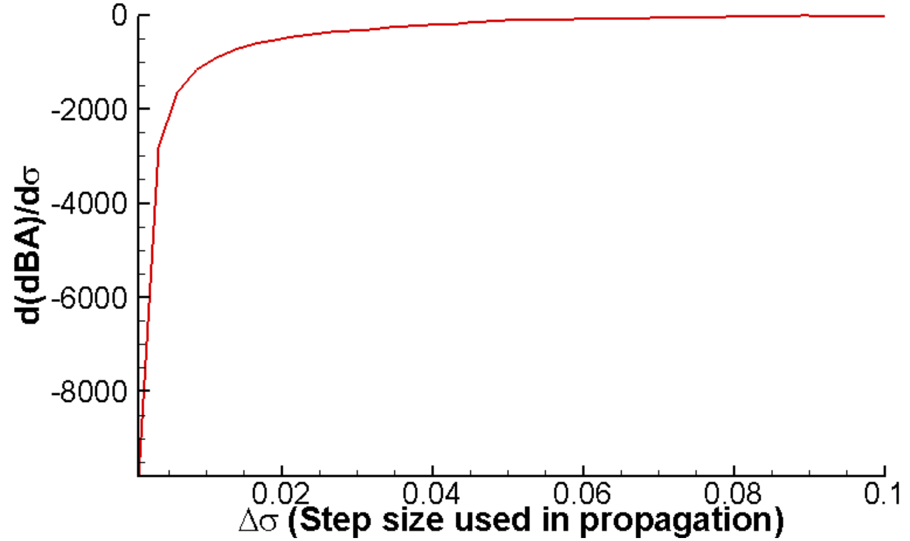


Sriram.Rallabhandi@nasa.gov

Results: Atmospheric Sensitivity

- Loudness sensitivity to number of points used during propagation asymptotically approaches zero
- Loudness is very sensitive to step size at extremely low step sizes
  - Quickly approaches zero for higher numbers





Discussion

- Adjoint-based design optimization used to generate targets
  - Targets generated in literature were based on either non-gradient approaches or linearized boom minimization theory
  - New approach provides an efficient way to generate targets
- Adjoint-based atmospheric sensitivity analysis
  - Some sources of epistemic uncertainties considered
  - Possible extension to include aleatory uncertainties such as temperature, winds, and relative humidity profiles
  - Quantify uncertainty during sonic boom propagation
  - Robust design point where error and sensitivity are simultaneously minimized
  - Use information to understand and improve design

Summary

- sBOOM framework extended to generate boom sensitivities with respect to equivalent areas, flight conditions, and propagation parameters
- Target equivalent areas generated using gradient-based optimization at multiple azimuthal angles
- Sensitivity of boom metrics to flight conditions and propagation parameters obtained, plotted and observations made

QUESTIONS?

Acknowledgments:

- NASA High Speed project
- Jim Fenbert for setting up the ModelCenter model for gradient-based optimization to generate target equivalent areas
- Karl Geiselhart (ModelCenter)
- Irian Ordaz (off-track analysis)
- Mathias Wintzer for manuscript review and general discussions
- Lori Ozoroski for manuscript review and support