Uncertainty Quantification and Certification Prediction of Low-Boom Supersonic Aircraft Configurations

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Outline

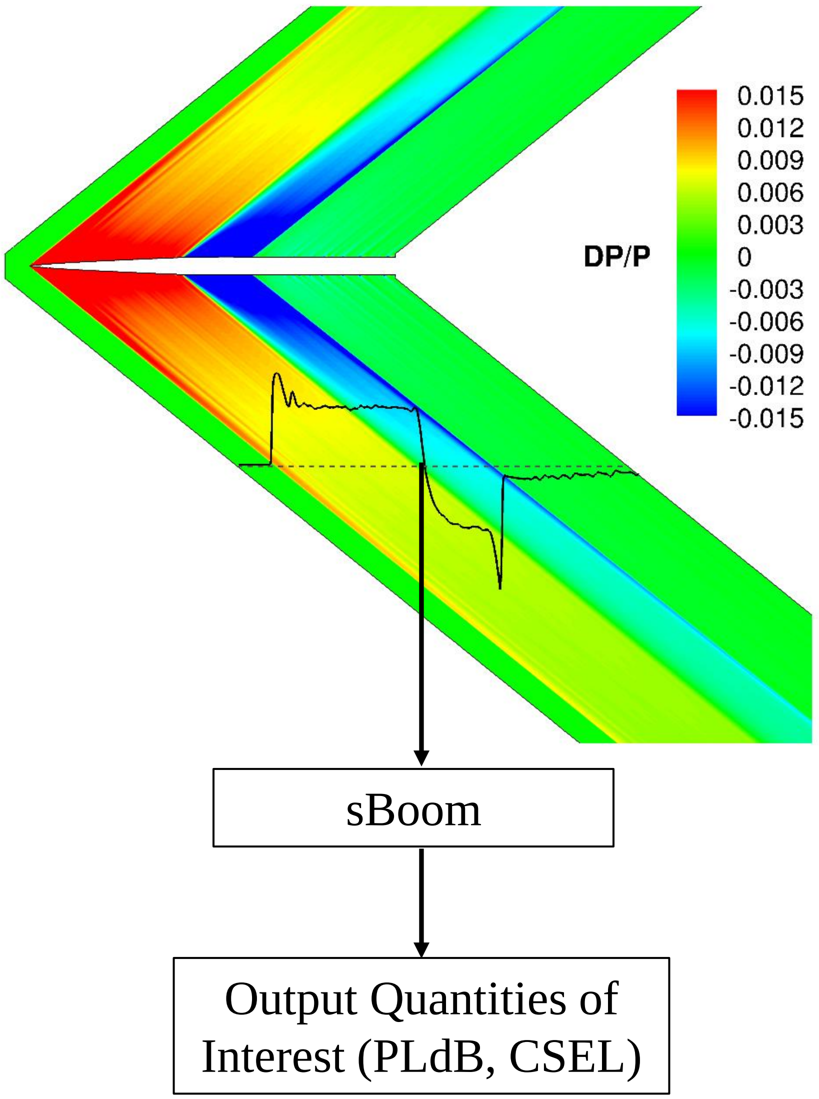
- Objectives and Motivation
- Computational Fluid Dynamics Approach for Boom Predictions
- Types of Uncertainty in Numerical Modeling
- Uncertainty Quantification and Sensitivity Analysis using Polynomial Chaos Expansions
- Certification Prediction Approach
- Demonstration on Sonic-Boom Configurations
- Conclusions

Objectives

- Develop a framework for efficient, accurate, scalable uncertainty quantification and certification prediction of sonic boom configuration models.
- Implement a nonintrusive, surrogate modeling approach based on polynomial chaos theory for efficient application to high-fidelity, multiphysics modeling.
- Determine the global nonlinear sensitivity of sonic boom measures to uncertain inputs using an approach based on the polynomial chaos expansion.
- Demonstrate the framework on three sonic boom configurations:
  - SEEB-ALR Body of Revolution
  - NASA 69^{o} Delta Wing
  - Lockheed Martin (LM) 1021-01 Low Boom Configuration

Complex Physics Models for Boom Prediction

- High fidelity approach for sonic boom propagation
  - Resolve near-field delta pressure with CFD
  - Propagate near-field signature to the ground with sBoom
  - Measure the uncertainty in quantities of interest (PLdB, CSEL)
- FUN3D
  - Fully Unstructured Navier-Stokes 3D flow solver
- Both Euler and fully turbulent cases were investigated
  - Fully turbulent cases used one equation Spalart-Allmaras model



Types of Uncertainty in Numerical Modeling

- Inherent (Aleatory) uncertainty
  - Inherent variation of a physical system (irreducible)
  - Represented mathematically with probability density function (PDF)
  - Examples – Freestream properties, manufacturing tolerances, etc.

- Epistemic uncertainty
  - Arises due to ignorance, lack of knowledge, or incomplete information (reducible)
  - Can be represented using intervals
  - Examples – Tunable modeling parameters, uncharacterized flight path conditions, turbulence model closure coefficients, etc.

Uncertainty Quantification and Sensitivity Analysis

- Uncertainty Quantification
  - Surrogate-based approach implemented for computational efficiency.
  - Surrogate developed using **Point-Collocation Nonintrusive Polynomial Chaos.**
  - Uncertainty propagated through the surrogate model using **second-order probability** for treatment of mixed (aleatory and epistemic) uncertainty.
  - Surrogate accuracy verified using test points.

- Sensitivity Analysis
  - Sensitivities obtained from **Sobol Index** approach.
  - Sobol indices are **based on the polynomial chaos expansion (PCE)** (no further CFD model evaluation).
  - Total Sobol indices are the **global nonlinear sensitivities** of the model to each uncertain parameter.

Basics of Polynomial Chaos (PC)

Spectral Representation of a Random Function or Response:

$$\alpha^*(\vec{x}, t, \vec{\xi}) \approx \sum_{j=0}^{P} \alpha_j(\vec{x}, t)\Psi_j(\vec{\xi})$$

Deterministic component — Random component

$\vec{\xi} = (\xi_1, ..., \xi_n)$ → n-dimensional independent random variable vector

$\Psi_j(\vec{\xi})$ → random basis functions (orthogonal polynomials i.e., Legendre polynomial if $\vec{\xi}$ is uniform and Hermite polynomials if $\vec{\xi}$ is normal)

$N_t = P + 1$ → total number of output modes

$$N_t = P + 1 = \frac{(n+p)!}{n!p!}$$

p : polynomial order of total expansion

Need to determine the expansion coefficients!

Point-Collocation Non-Intrusive PC

- One approach for determining the coefficients of the PC expansion is to use a point-collocation approach.
- For a given PC of order p and n random dimensions, choose N_S sample points to evaluate the deterministic model.
- Solve a linear system for the modes.

$$\begin{pmatrix} \Psi_0(\xi_0) & \Psi_1(\xi_0) & \cdots & \Psi_P(\xi_0) \\ \Psi_0(\xi_1) & \Psi_1(\xi_1) & \cdots & \Psi_P(\xi_1) \\ \vdots & \vdots & \ddots & \vdots \\ \Psi_0(\xi_{N_s}) & \Psi_1(\xi_{N_s}) & \cdots & \Psi_P(\xi_{N_s}) \end{pmatrix} \begin{pmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_P \end{pmatrix} = \begin{pmatrix} \alpha^*(x,\xi_0) \\ \alpha^*(x,\xi_1) \\ \vdots \\ \alpha^*(x,\xi_{N_s}) \end{pmatrix}$$

$$\left(N_S \ \text{x} \ N_t\right)$$

- For an overdetermined system ($N_S > N_t$), use a Least Squares approach to obtain the modes.

Global Non-Linear Sensitivity Analysis with Sobol Indices

- Objective: Rank the relative importance of each input uncertain variable to the overall output uncertainty using non-linear global sensitivity analysis.

$$S_{i_1 \cdots i_s} = \frac{D_{i_1,\ldots,i_s}}{D}$$

$\Rightarrow$ Partial variance (calculated from PCE)

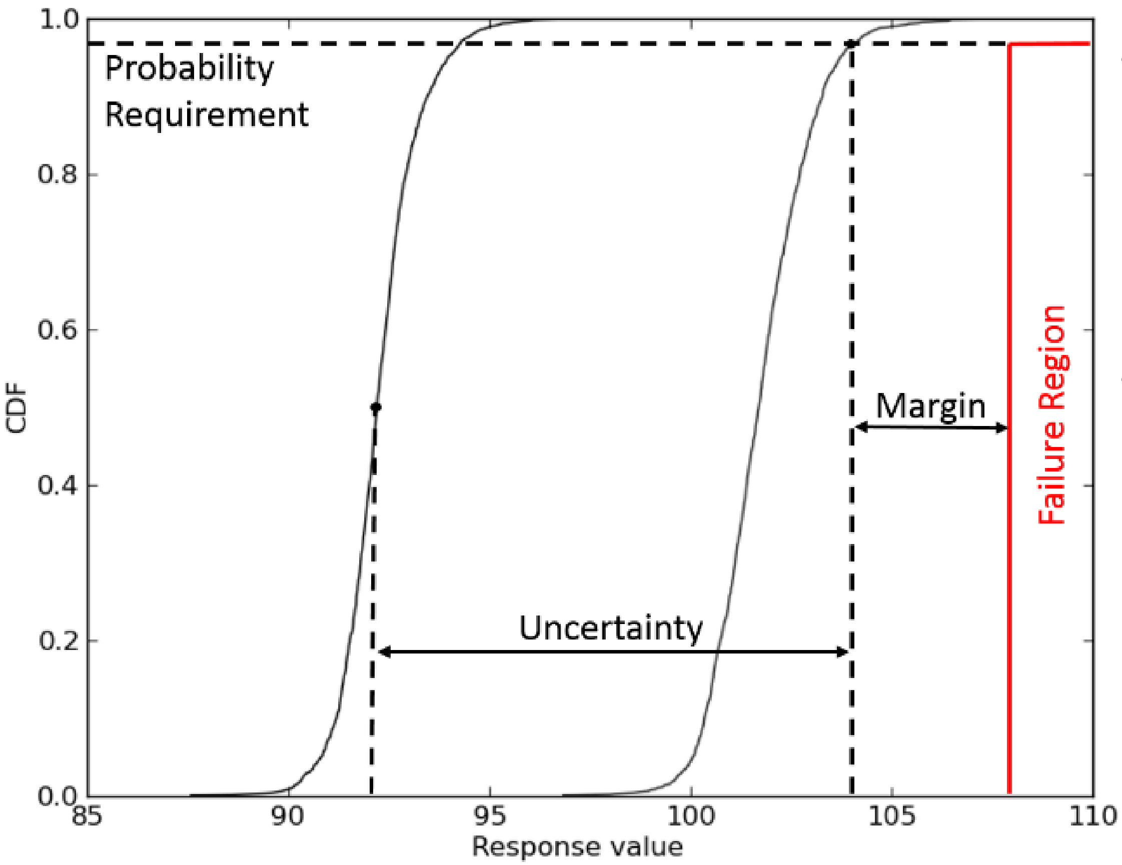
$\Rightarrow$ Total variance (calculated from PCE)

$$D_{i_1,\ldots,i_s} = \sum_{\beta \in \{i_1,\ldots,i_s\}} \alpha_\beta^2(t,\vec{x}) \left\langle \Psi_\beta^2(\vec{\xi}) \right\rangle , \qquad 1 \le i_1 < \ldots < i_s \le n$$

$$D = \sum_{j=1}^{P} \alpha_j^2(t,\vec{x}) \left\langle \Psi_j^2(\vec{\xi}) \right\rangle$$

Certification Prediction Approach

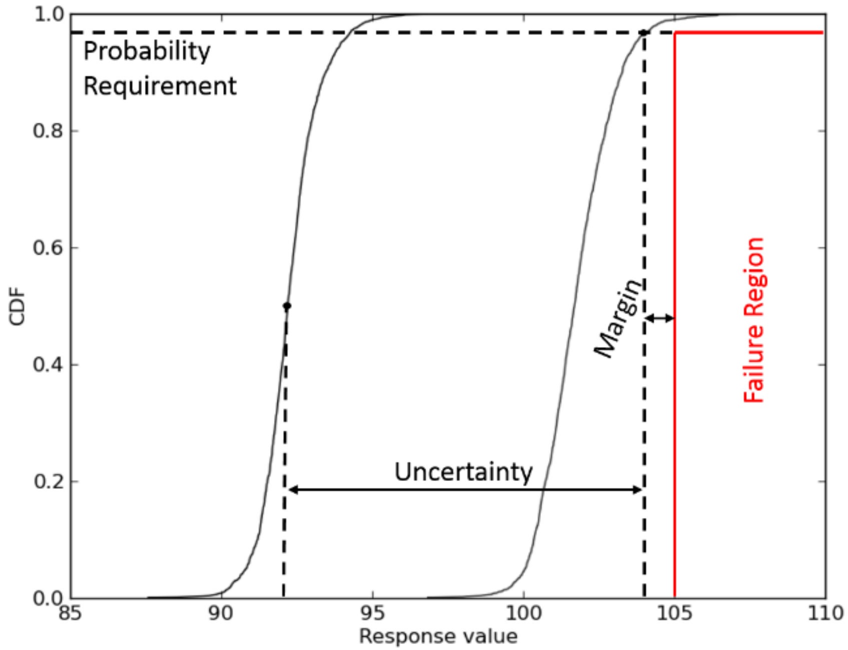
- Using the UQ results, a method known as the quantification of margins and uncertainties (QMU) can be used to measure the confidence in a design.
- QMU compares the uncertainty in both the design and some threshold with a margin between the two using a confidence ratio.



- Using a Probability box representation of the uncertainty, a comparison can be made between the output region and the certification value.
- Analysis done at a specified probability level

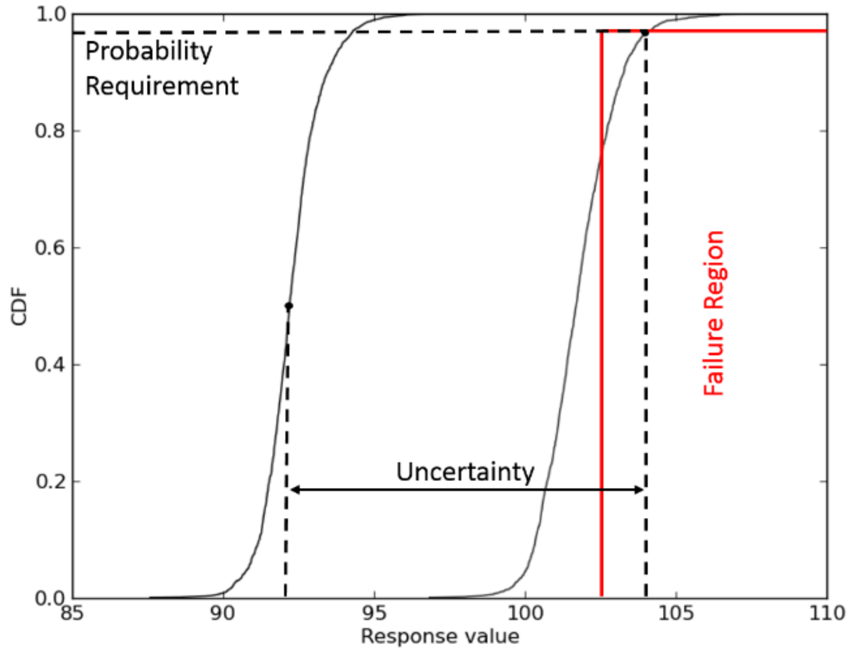
$$\text{Confidence Ratio} = \frac{\text{Margin}}{\text{Uncertainty}}$$

Certification Prediction Approach



Small Positive Margin

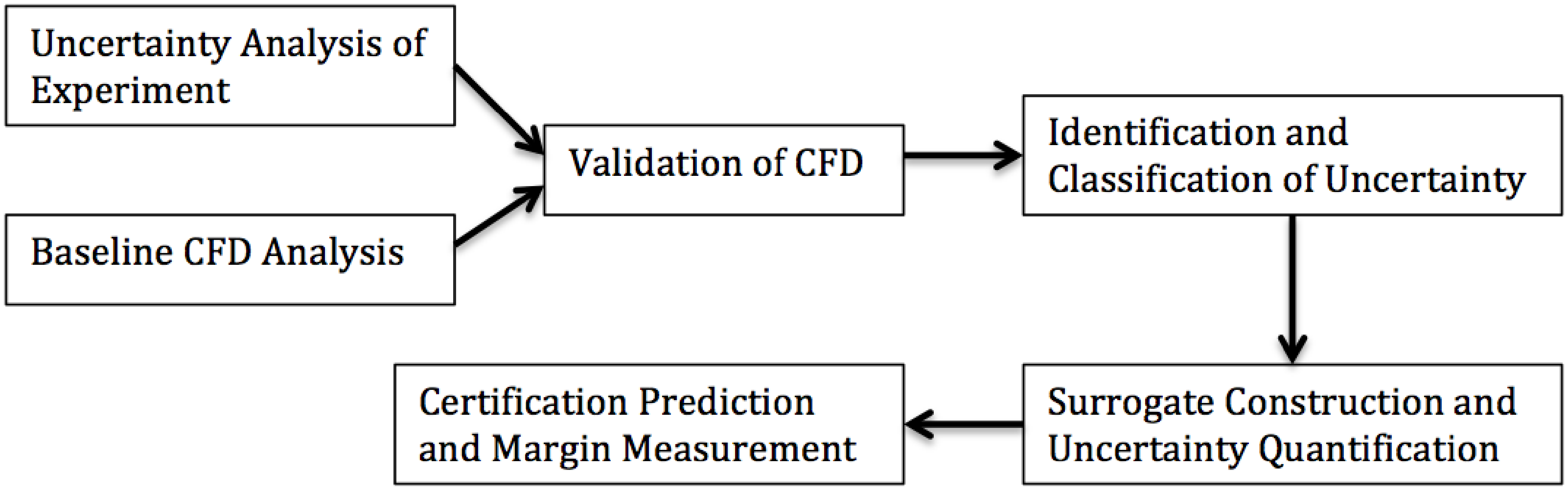
- Margin may be small with respect to the uncertainty
- Indication of weak reliability that the design may pass certification.



Negative Margin

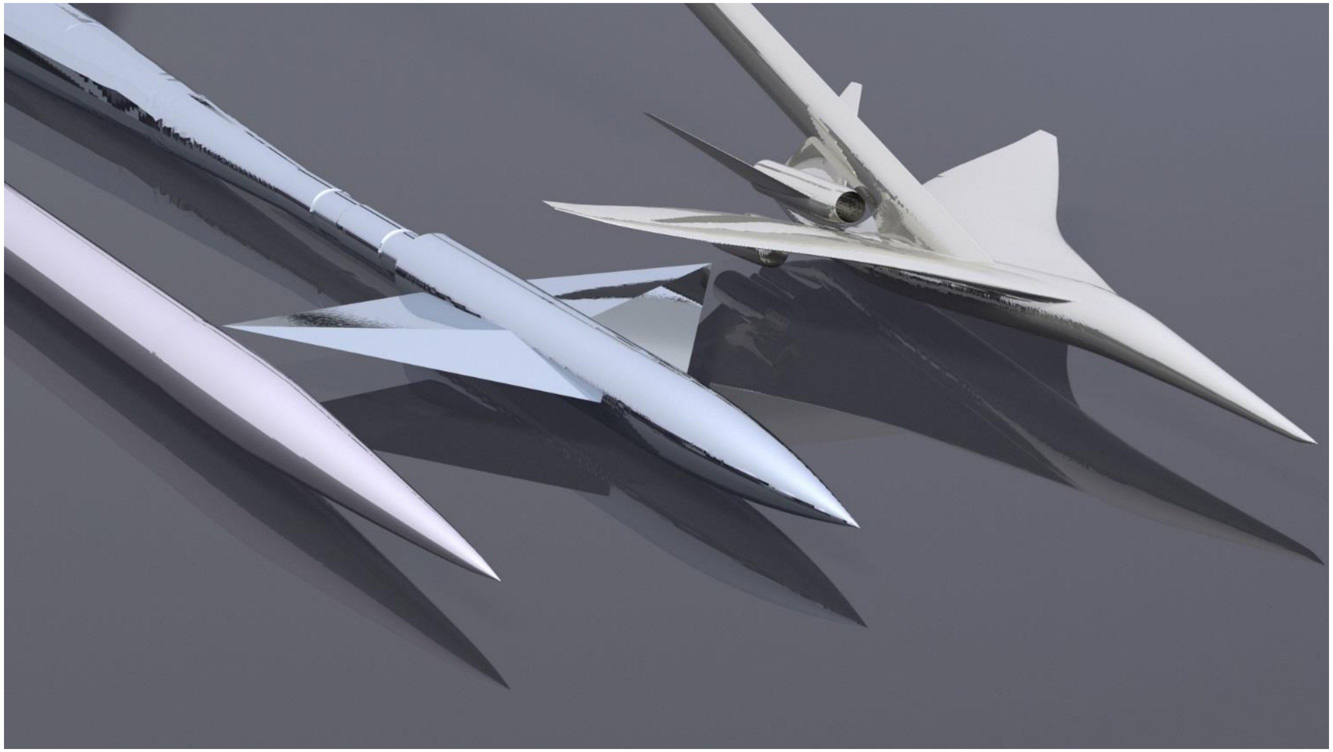
- Failure region may cross into the output probability region.
- Certification prediction unlikely.

UQ and Certification Prediction Process Summary

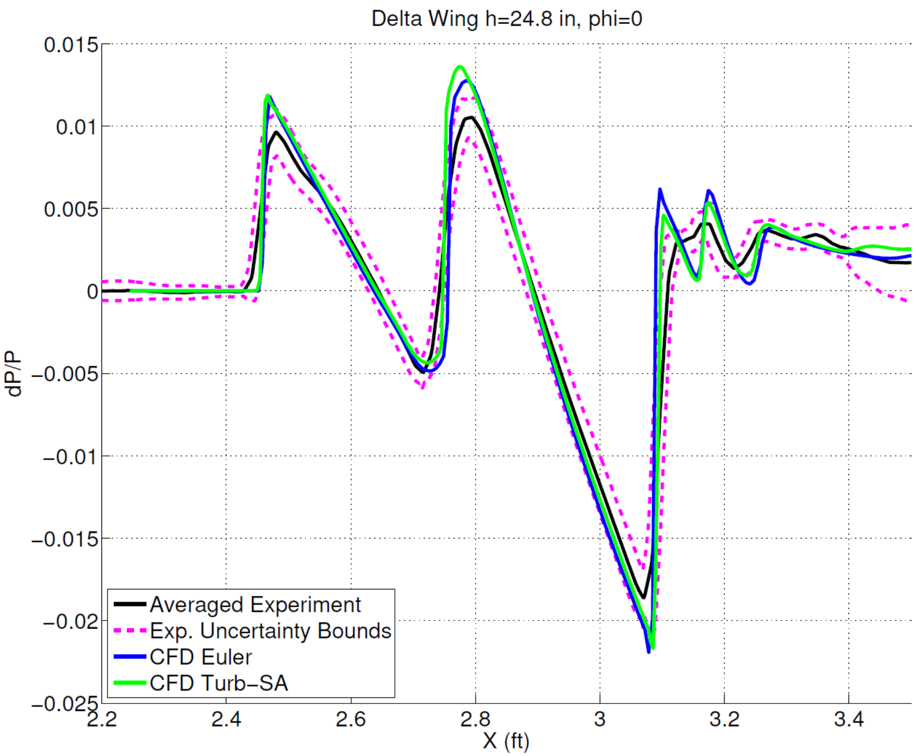


Low-Boom Configurations

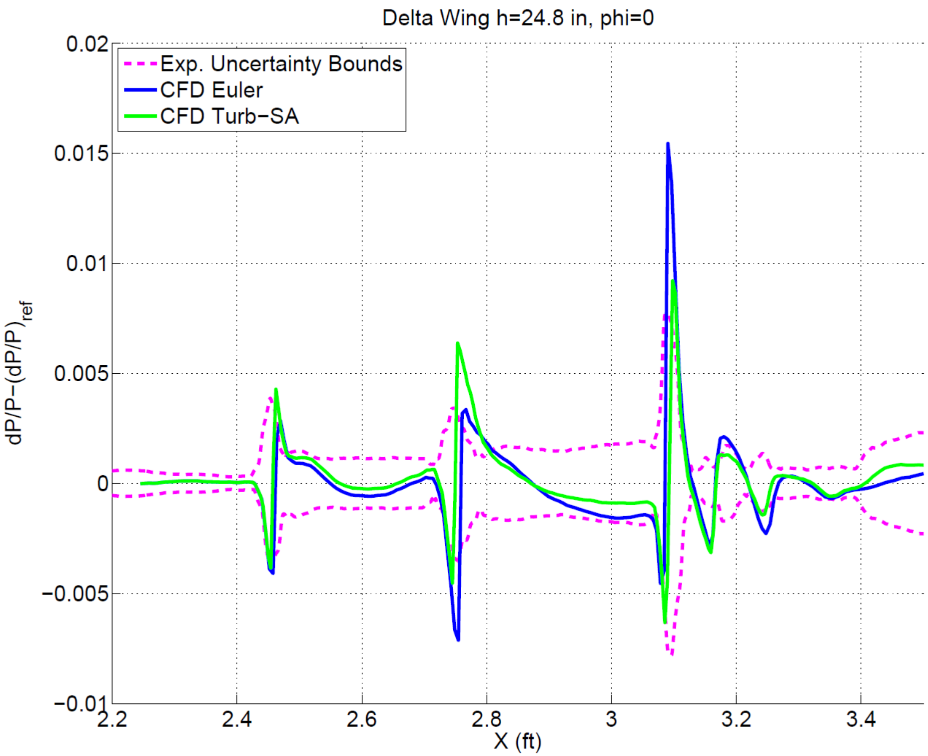
- SEEB-ALR
  - As-built and As-designed
- NASA 69 deg. Delta Wing
- Lockheed Martin 1021-01 Configuration



69º Delta Wing: Comparison with Experiment



Actual Signatures



Residual Scale

Uncertain Input Parameters

CFD Aleatory Inputs

Input	Distribution	Mean	Std. Dev.
Angle of Attack	Gaussian	0.0	0.1
Mach Number	Gaussian	1.6/1.7	0.0016

sBoom Aleatory Inputs

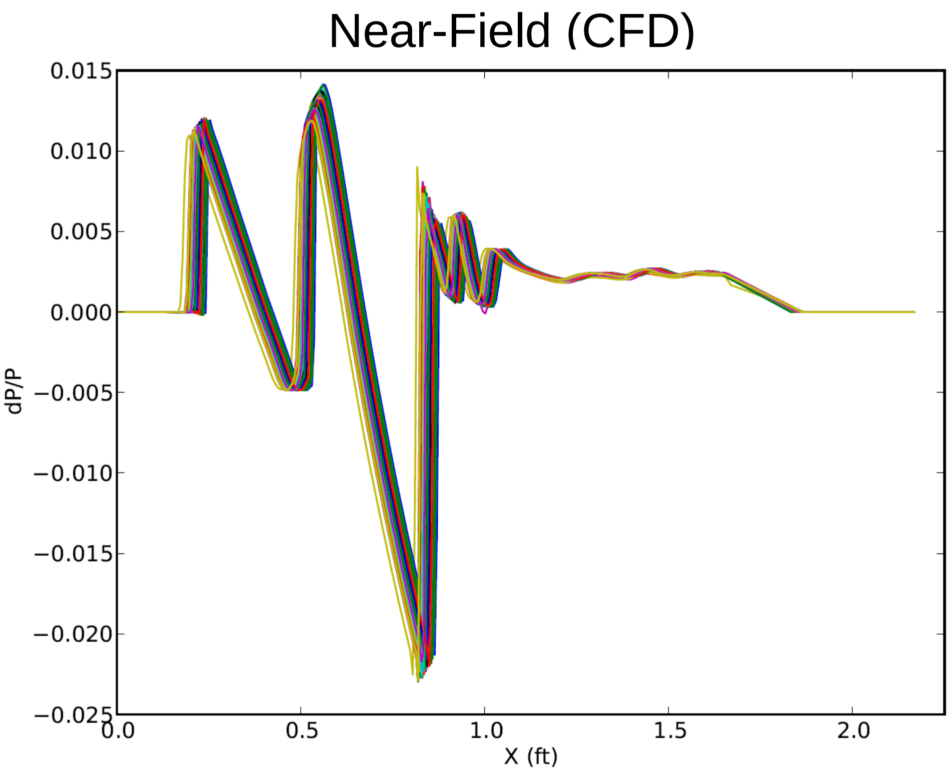
Input	Distribution	Mean	Std. Dev.
Temperature Profile (%)	Gaussian	1.0	0.01
Humidity Profile (%)	Gaussian	1.0	0.01
Climb Angle (Deg.)	Gaussian	0.0	0.1
Azimuth (Deg.)	Gaussian	0.0	0.1
Turn Rate (Deg./s)	Gaussian	0.0	0.05
Climb Rate (Deg./s)	Gaussian	0.0	0.05

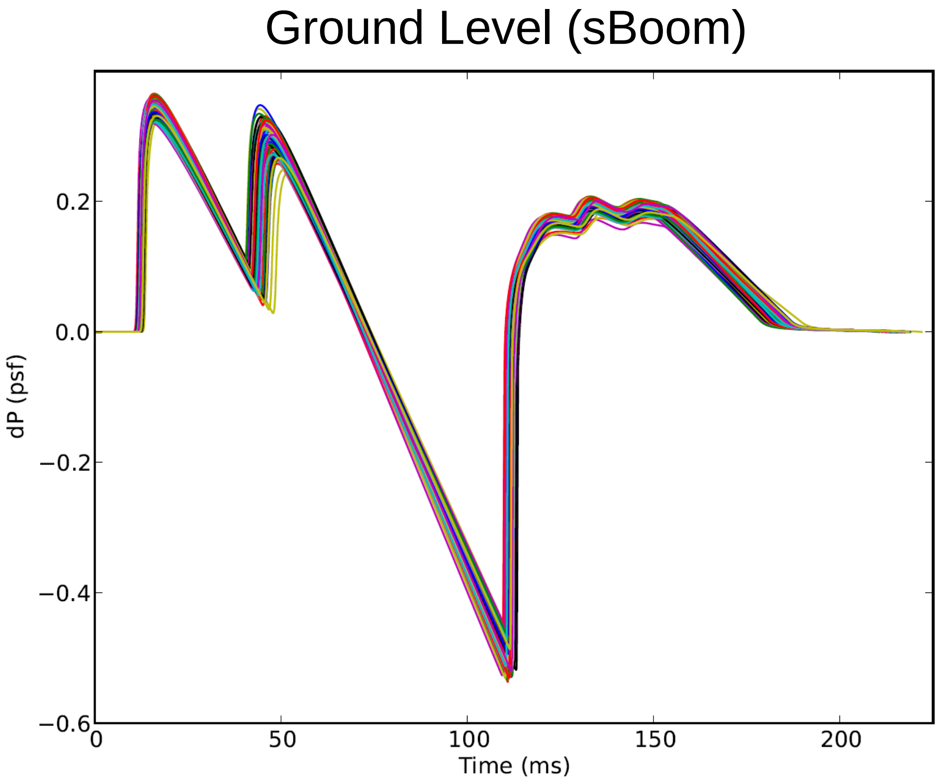
sBoom Epistemic Inputs

Input	Min.	Max.
Initial Step Size	0.007	0.03
Reflection Factor	1.8	2.0
Ground Elevation (ft)	0.0	5000.0
Signature Propagation Points	20000	60000

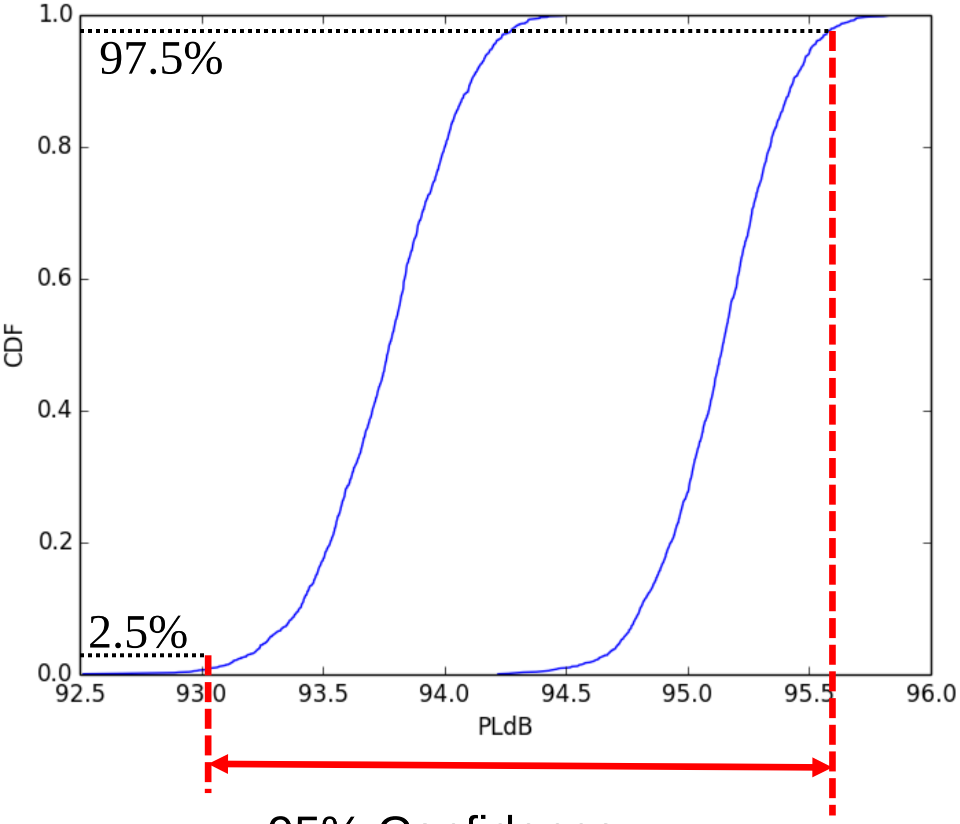
- Uncertainty exists in both the near-field CFD model and sBoom.
- Uncertain parameter information based on author discussion and expert opinion.
- These are the final values. Intermediate results were used to improve the results.

69^{o} Delta Wing Euler:
182 Deterministic Model Samples (2nd Order PCE)

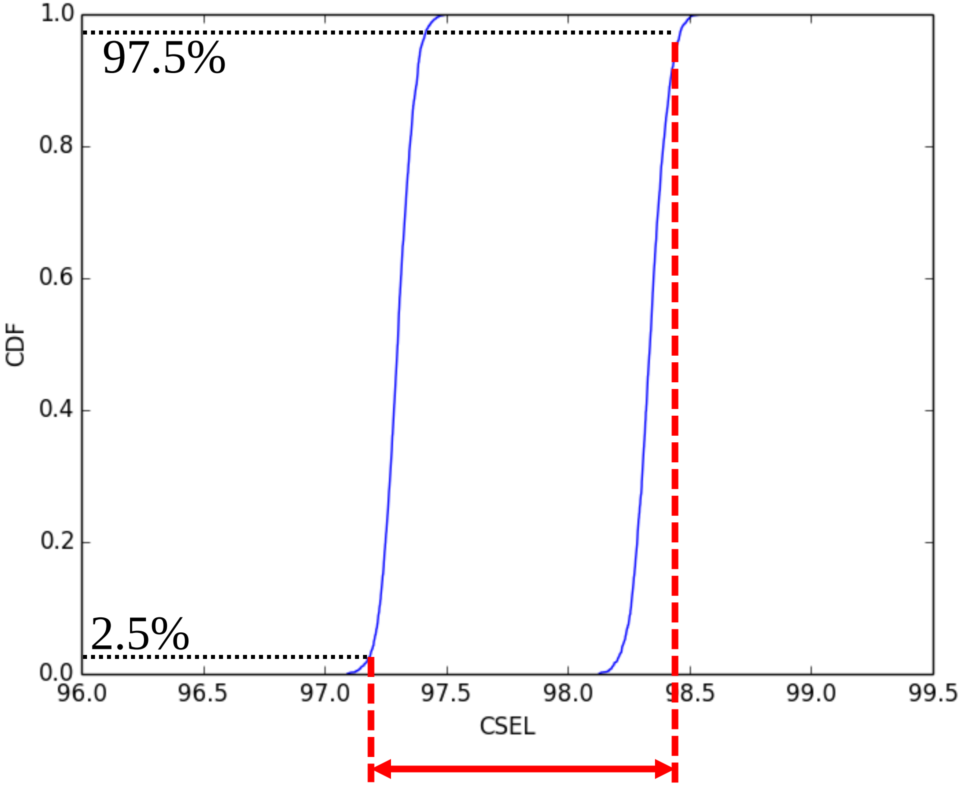




69º Delta Wing Euler: Probability-Box Output Representation

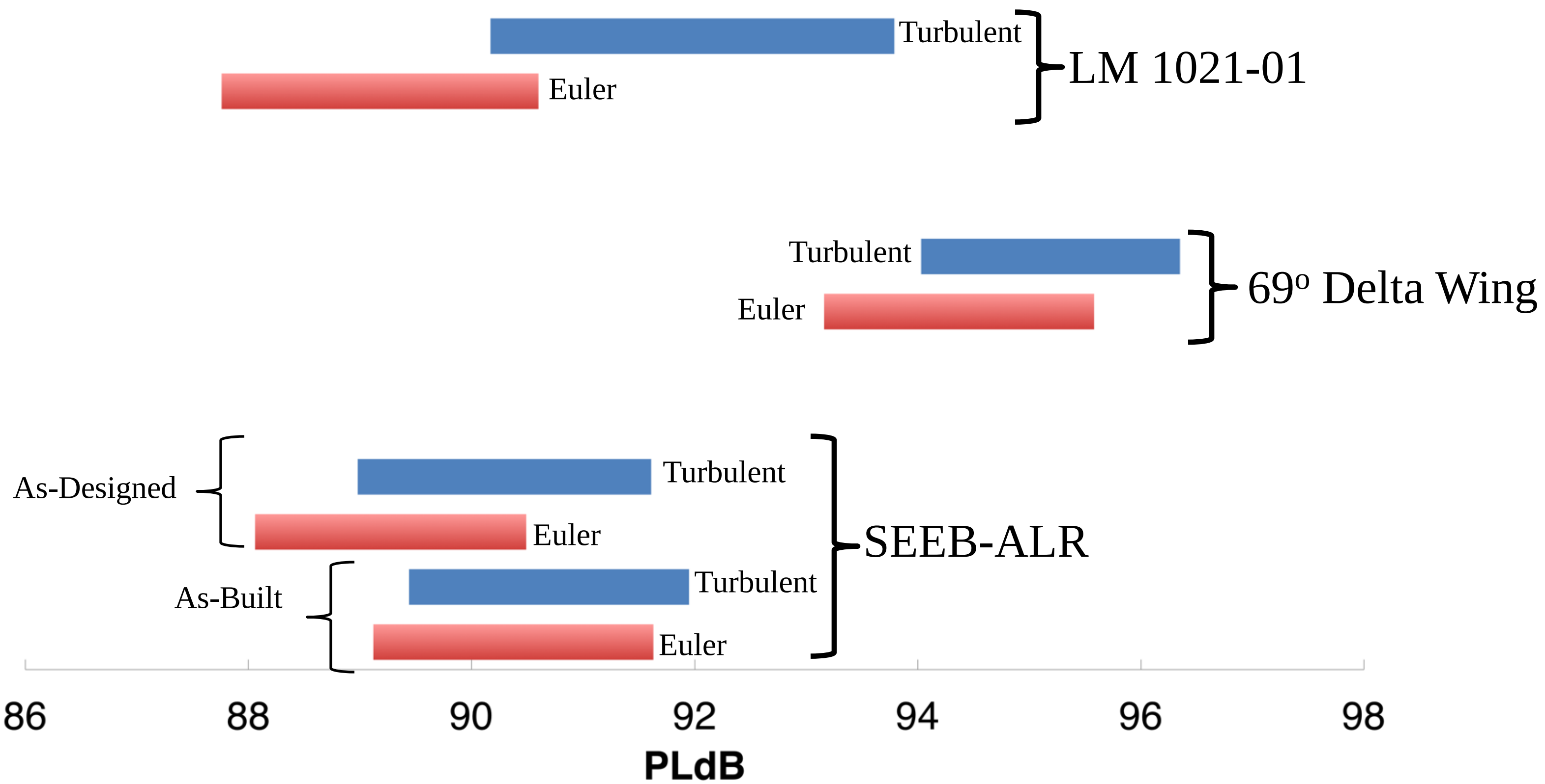


95% Confidence
Interval = [93.16 , 95.58]



95% Confidence
Interval = [97.18 , 98.46]

Sonic Boom Configuration Summary: PLdB 95% Confidence Intervals



Sonic Boom Configuration Summary: Global Nonlinear Sensitivities via Sobol Indices

Variable Contribution to PLdB greater than 10%

SEEB-ALR

Uncertain Parameter	Euler as-Built	Euler as-Designed	Turbulent as-Built	Turbulent as-Designed
Reflection Factor	46.4%	44.8%	45.9%	44.2%
Humidity Profile	38.3%	35.7%	41.6%	36.1%

69º Delta Wing

Uncertain Parameter	Euler	Turbulent
Reflection Factor	50.9%	52.0%
Humidity Profile	37.1%	38.0%

LM 1021-01

Uncertain Parameter	Euler	Turbulent
Reflection Factor	33.8%	21.9%
Humidity Profile	22.7%	17.9%
Angle of Attack	39.0%	55.1%

Angle of Attack becomes import due to LM 1021-01 design features.

- For CSEL, humidity profile contribution drops below 10%.
- Reflection factor dominates for SEEB-ALR and Delta Wing.
- Angle of attack still important for LM 1021-01.

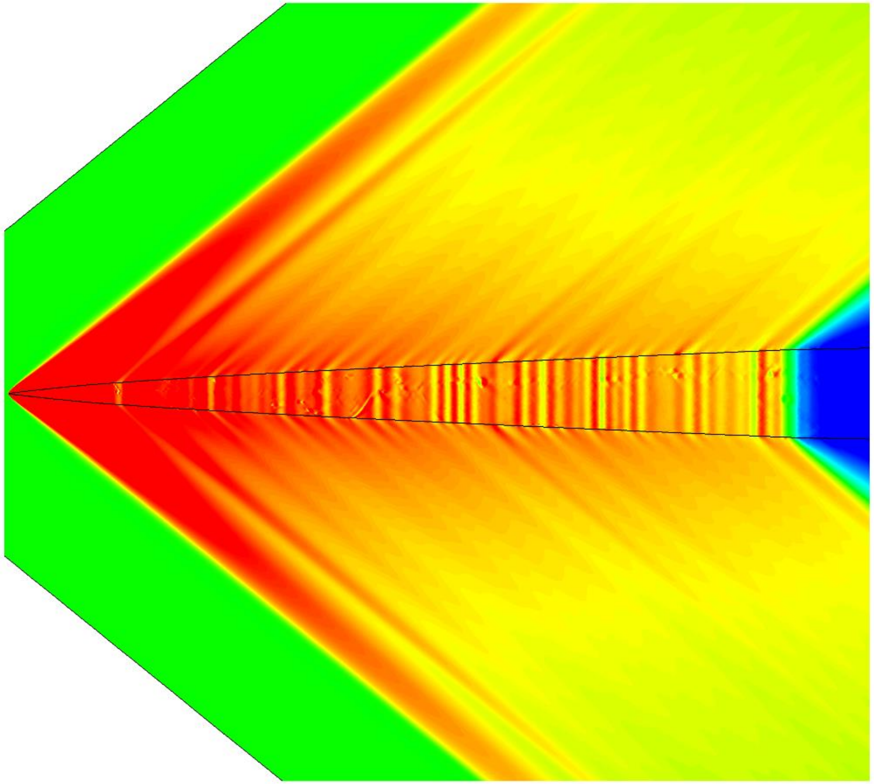
Final Remarks

- Developed an efficient, accurate, and scalable framework for uncertainty quantification and certification prediction of low-boom configurations.
- Implemented a nonintrusive, surrogate modeling approach based on polynomial chaos theory for efficient application to high-fidelity multiphysics modeling.
- Determined the global nonlinear sensitivity of low-boom measures to uncertain inputs using an approach based on the polynomial chaos expansion.
- Demonstrated the framework on three sonic-boom configurations:
    - SEEB-ALR Body of Revolution (as-built and as-designed)
    - NASA 69^{o} Delta Wing
    - Lockheed Martin (LM) 1021-01 Low Boom Configuration

Questions?

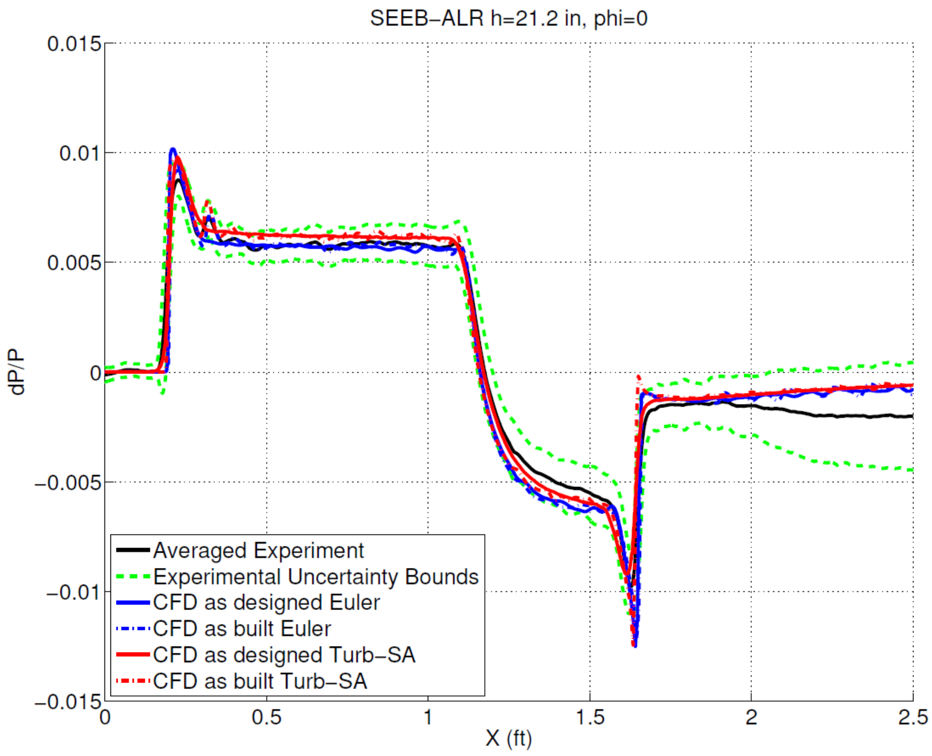
Backup

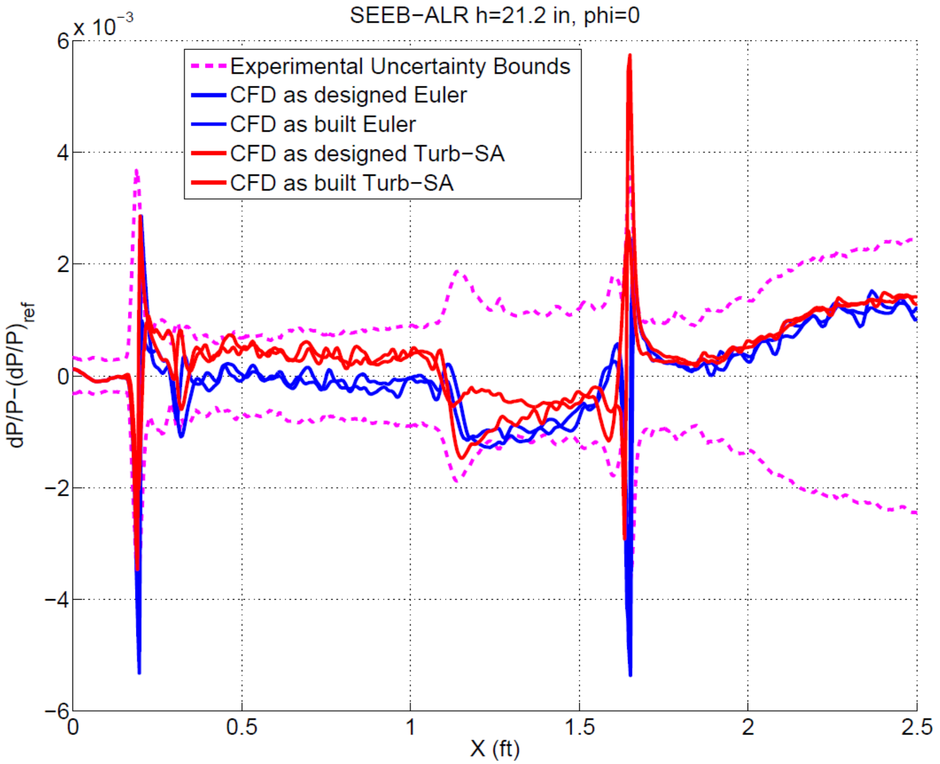
SEEB-ALR as-Build vs. as-Designed



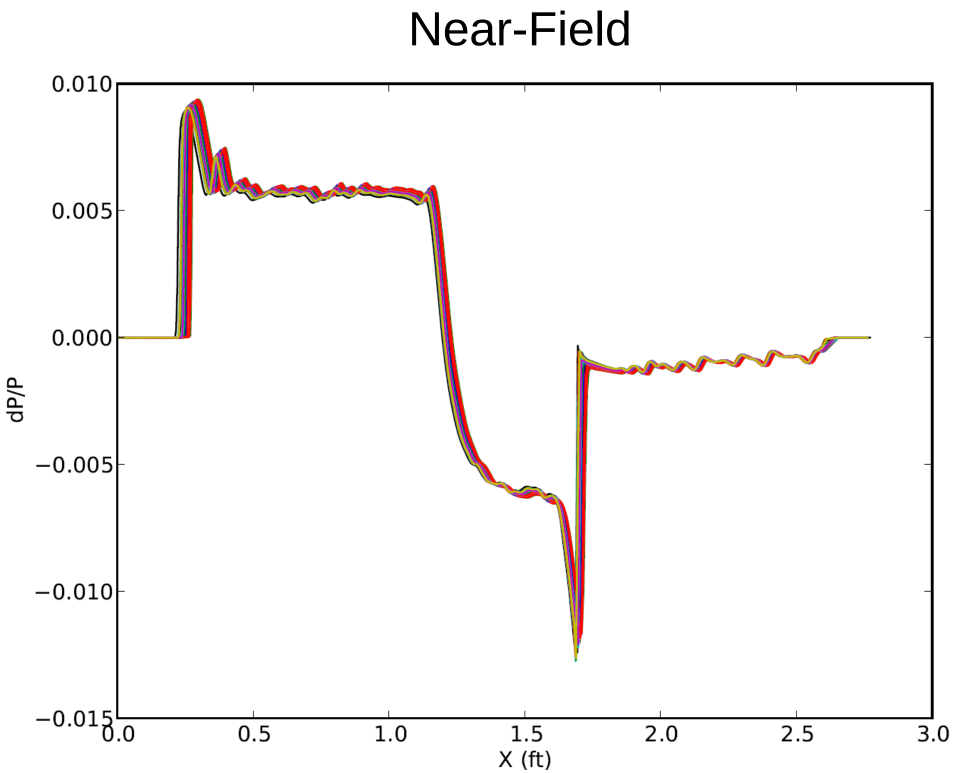
- Noticeable surface imperfections of the as-build SEEB-ALR model.
- CFD model detects these features and they are propagated to the ground level.

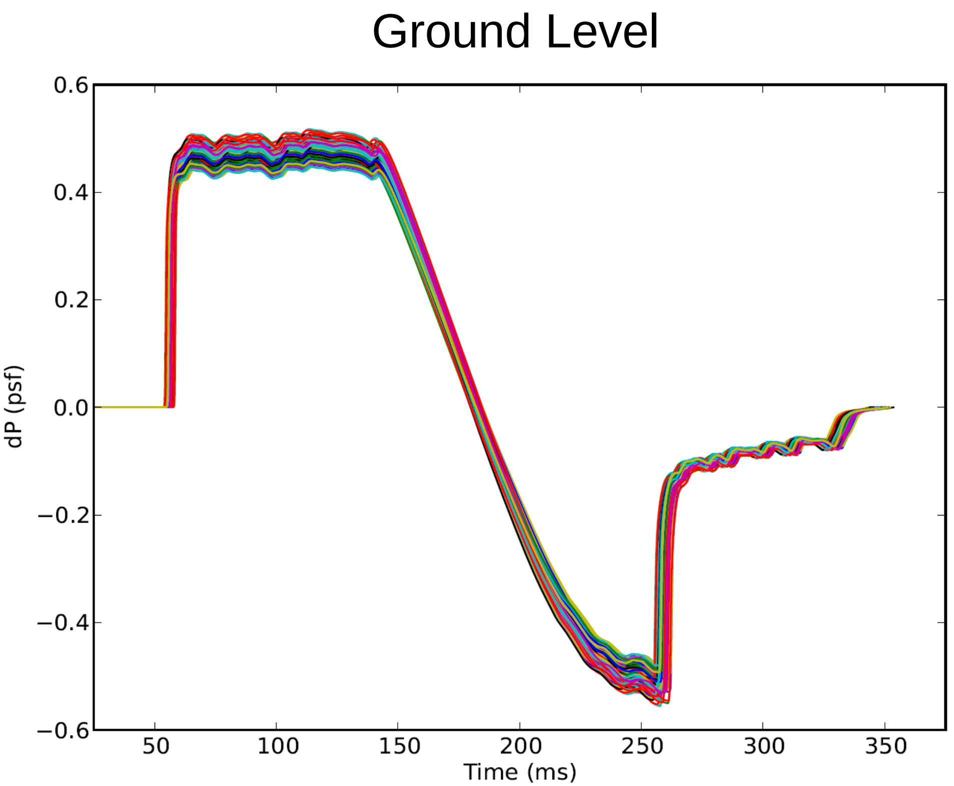
SEEB-ALR:
Comparison with Experiment





SEEB-ALR Euler as-built: 182 Deterministic Model Samples (2nd Order PCE)





SEEB-ALR Euler as-built : Global Nonlinear Sensitivities via Sobol Indices

Contribution to PLdB

Uncertain Parameter	Euler as-Built	Euler as-Designed	Turbulent as-Built	Turbulent as-Designed
Angle of Attack	4.7%	9.6%	2.4%	6.7%
Initial Step Size	1.6%	1.1%	1.7%	1.8%
Reflection Factor	46.4%	44.8%	45.9%	44.2%
Humidity Profile	38.3%	35.7%	41.6%	36.1%
Ground Elevation	7.9%	7.7%	6.8%	9.7%
All Others	<1%	<1%	<1%	<1%

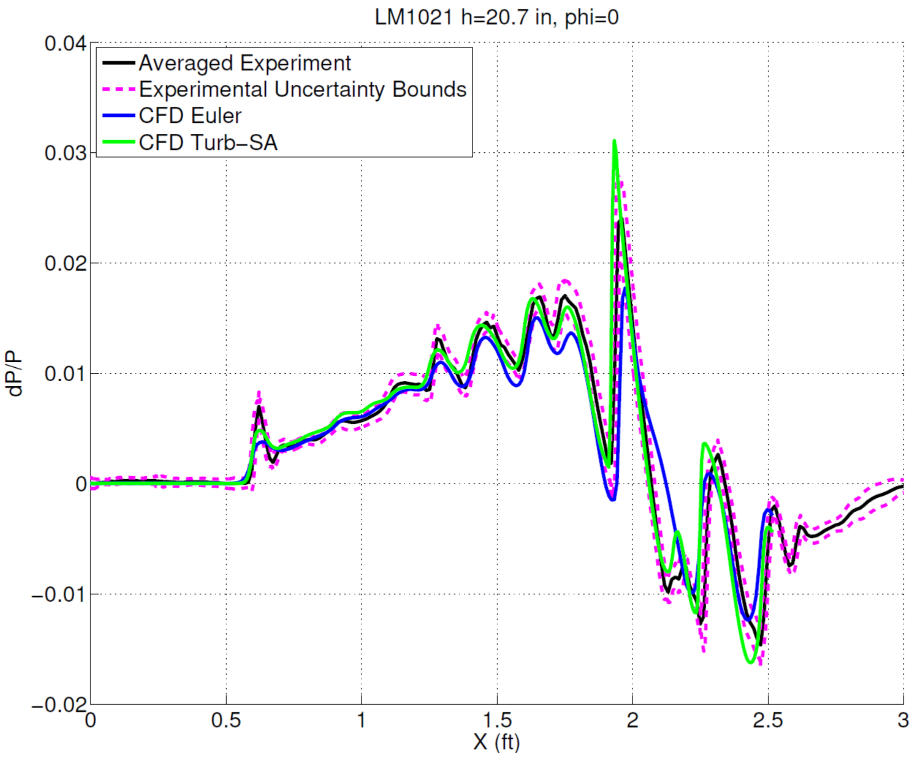
Largest Contributors

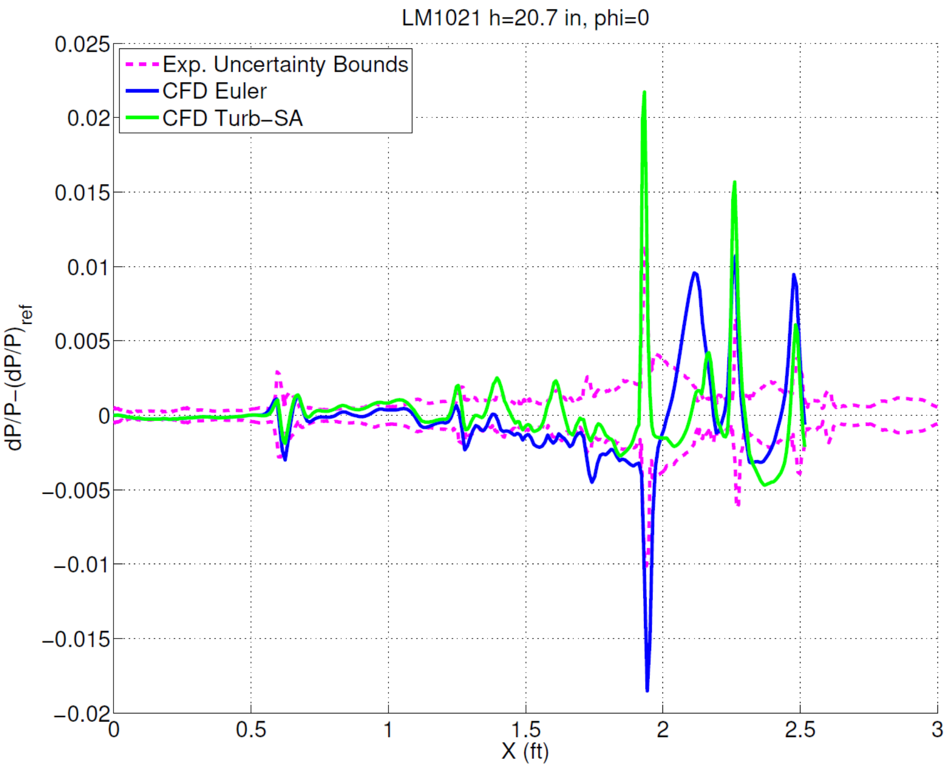
Contribution to CSEL

Uncertain Parameter	Euler as-Built	Euler as-Designed	Turbulent as-Built	Turbulent as-Designed
Angle of Attack	3.6%	6.2%	4.5%	4.6%
Reflection Factor	88.2%	84.1%	86.5%	86.0%
Temperature Profile	2.2%	2.4%	2.4%	2.4%
Humidity Profile	1.7%	1.5%	1.7%	1.7%
Ground Elevation	4.1%	5.5%	4.6%	5.2%
All Others	<1%	<1%	<1%	<1%

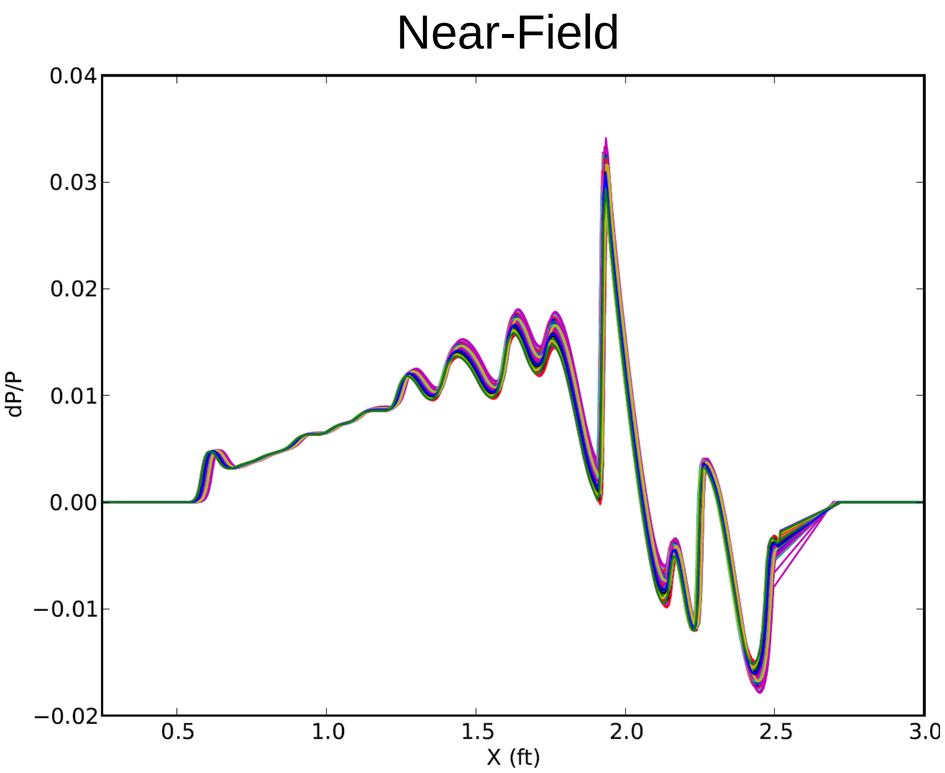
Largest Contributor

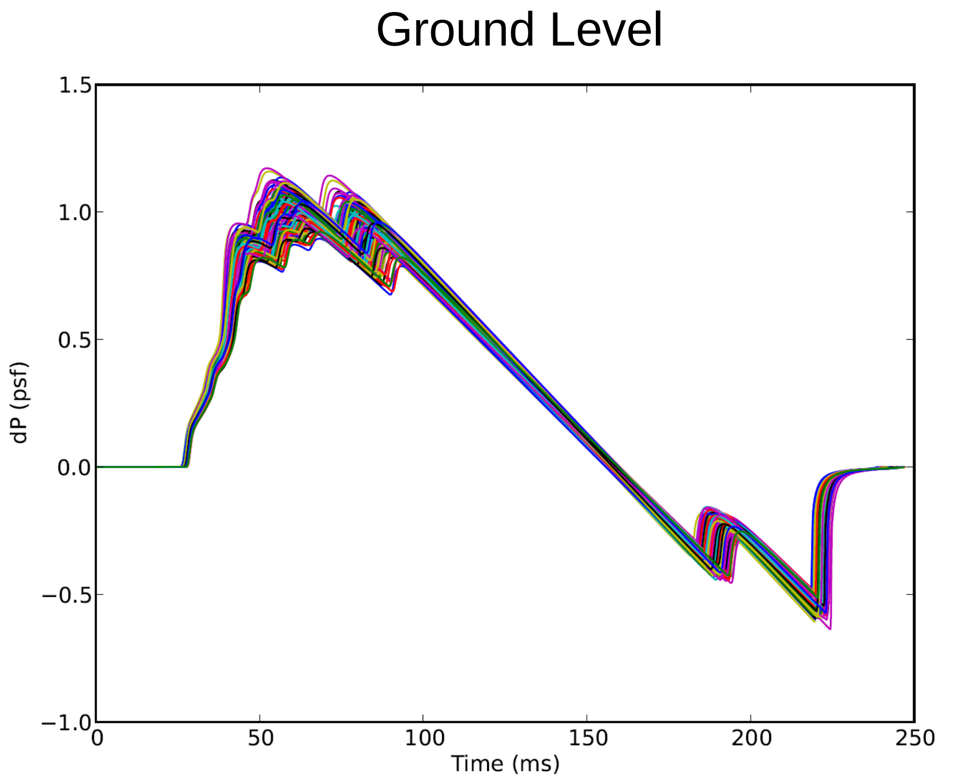
LM 1021-01 Euler: Comparison with Experiment





LM 1021-01 Euler:
182 Deterministic Model Samples (2nd Order PCE)





Low-Boom Configurations Summary: PLdB and CSEL 95% Confidence Intervals

SEEB-ALR

Configuration	PLdB	CSEL
Euler as-Built	[89.12 , 91.63]	[94.64 , 96.05]
Euler as-Designed	[88.06 , 90.49]	[94.32 , 95.80]
Turbulent as-Built	[89.44 , 91.95]	[94.78 , 96.22]
Turbulent as-Designed	[88.98 , 91.61]	[94.75 , 96.20]

69^{o} Delta Wing

Configuration	PLdB	CSEL
Euler	[93.16 , 95.58]	[97.18 , 98.46]
Turbulent	[94.03 , 96.35]	[97.63 , 98.85]

LM 1021-01

Configuration	PLdB	CSEL
Euler	[87.76 , 90.60]	[94.43 , 96.85]
Turbulent	[90.17 , 93.79]	[96.06 , 98.76]

Sonic Boom Configuration Summary: Global Nonlinear Sensitivities via Sobol Indices

Contribution to PLdB

Uncertain Parameter	**Euler**	**Turbulent**
Initial Step Size	1.4%	1.0%
Reflection Factor	50.9%	52.0%
Temperature Profile	1.3%	1.8%
Humidity Profile	37.1%	38.0%
Ground Elevation	7.9%	6.3%
All Others	<1%	<1%

Largest Contributors

Contribution to CSEL

Uncertain Parameter	**Euler**	**Turbulent**
Reflection Factor	93.1%	94.4%
Temperature Profile	2.1%	2.5%
Humidity Profile	1.1%	1.5%
Ground Elevation	1.9%	1.4%
All Others	<1%	<1%

Largest Contributor

Sonic Boom Configuration Summary: Global Nonlinear Sensitivities via Sobol Indices

Variable Contribution to CSEL greater than 10%

SEEB-ALR

Uncertain Parameter	Euler as-Built	Euler as-Designed	Turbulent as-Built	Turbulent as-Designed
Reflection Factor	88.2%	84.1%	86.5%	86.0%

69^{o} Delta Wing

Uncertain Parameter	Euler	Turbulent
Reflection Factor	93.1%	94.4%

LM 1021-01

Uncertain Parameter	Euler	Turbulent
Reflection Factor	33.8%	21.9%
Angle of Attack	39.0%	55.1%

Angle of Attack becomes import due to LM 1021-01 design features.

Basics of PC

- The objective of the PC based methods is to calculate the coefficients in the stochastic expansion:

$$\alpha^*(\vec{x}, t, \vec{\xi}) \approx \sum_{j=0}^{P} \alpha_j(\vec{x}, t)\Psi_j(\vec{\xi})$$

- Various statistics can be obtained with the use of coefficients and the basis functions in the expansion

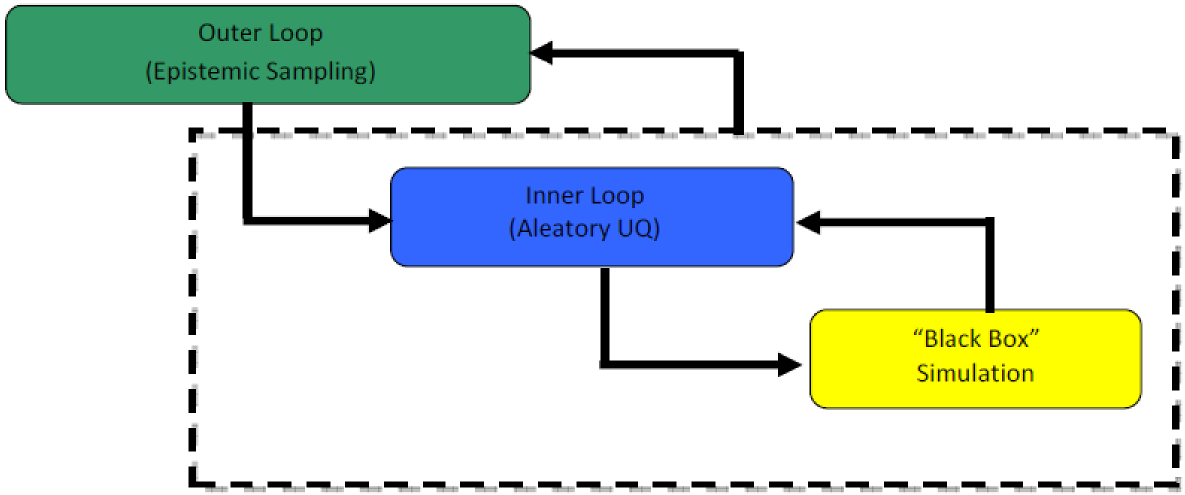
$$E_{PC}\left[\alpha^*(\vec{x}, t, \vec{\xi})\right] = \alpha_0(\vec{x}, t)$$

$$Var_{PC}\left[\alpha^*(\vec{x}, t, \vec{\xi})\right] = \sum_{j=1}^{P} [\alpha_j^2(\vec{x}, t) < \Psi_j^2 >]$$

- Two main approaches to calculate the coefficients
  - Intrusive PC
  - Non-Intrusive PC (NIPC)

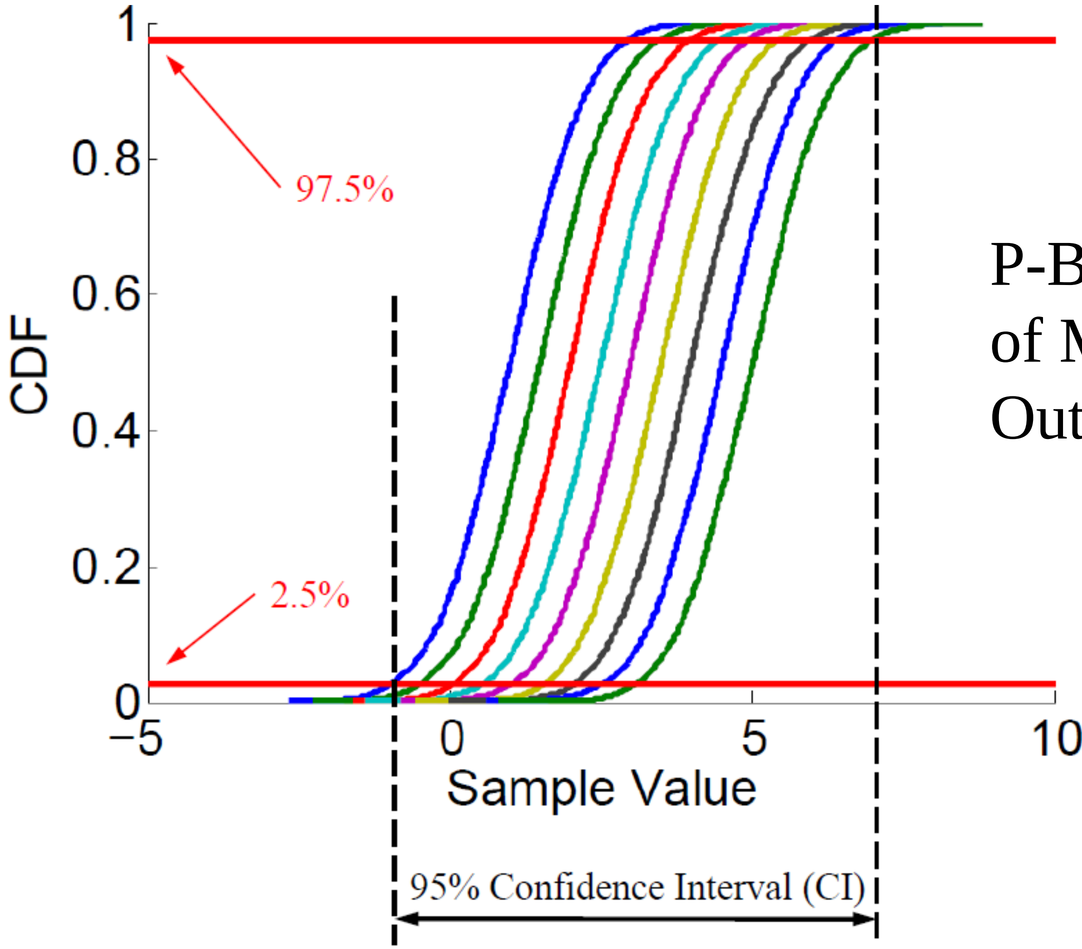
Analysis Under Mixed Uncertainty

- Second Order Probability Approach



- This analysis type can be computationally expensive when using traditional sampling techniques such as Monte Carlo.
- Epistemic loop can be analyzed using sampling or optimization.
- The approach in this study will be to replace the “Black Box” model with the NIPC response surface, which is a polynomial.

An Approach to Calculate 95% CI for Mixed UQ

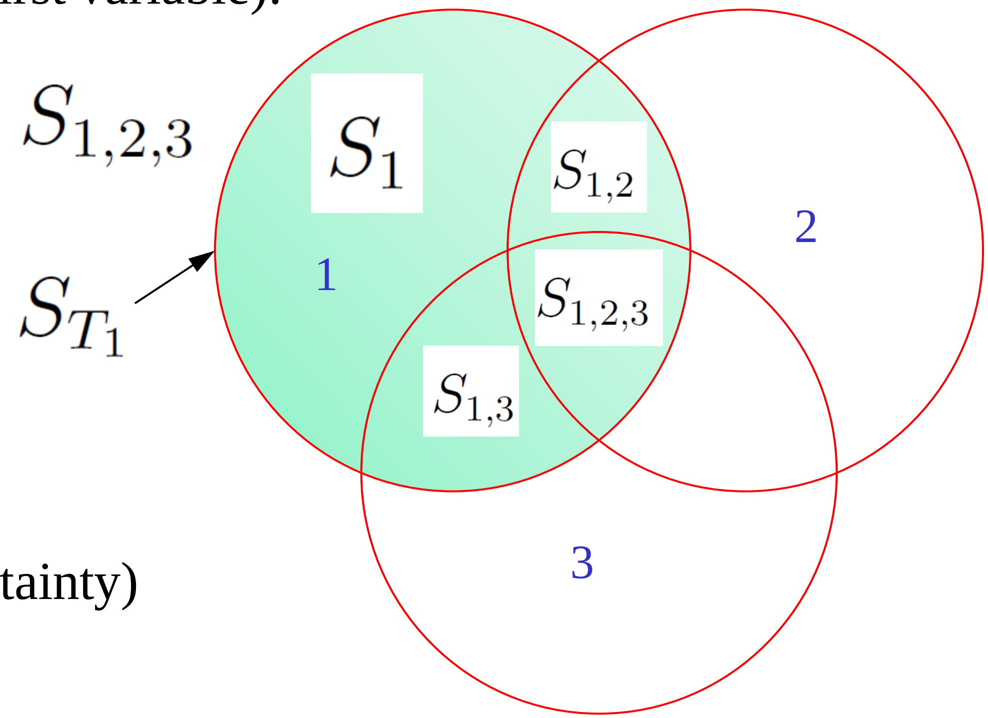


P-Box Representation of Mixed Uncertainty Output

Global Sensitivity Analysis with Sobol Indices (Cont.)

- Total indices
  - Summation of all the partial indices that include the particular parameter, e.g., n=3, i=1 (first variable):

$$S_{T_1} = S_1 + S_{1,2} + S_{1,3} + S_{1,2,3}$$



- Can use total indices to rank the importance (contribution to uncertainty) of each variable